

ST-2015

PAPER CODE : 4330

PAPER – ST- 501

Probability and Distributions

TIME: $2\frac{1}{2}$ Hours

4330

Marks: 70

INSTRUCTIONS: 1) There are 5 compulsory questions in this question paper.

2) All questions carry equal marks.

3) Use of Scientific calculator is allowed.

4) Statistical tables and graph paper will be provided on request.

- Q1 a) State and prove weak Law of Large Number. 7
b) Examine if the following sequence of random variable $\{X_k\}$, holds C.L.T. or not? 7

$$\text{Prob. } \{ X_k = \pm k^\alpha \} = \frac{1}{2} k^{-2\alpha} \text{ and Prob. } \{ X_k = 0 \} = 1 - k^{1-2\alpha}, \alpha < \frac{1}{2}$$

OR

- Q1 a) State and prove Chebyshev's Inequality for a continuous random variable. Give the uses of Chebyshev's inequality. 8
b) Check whether for the sequence of random variable $\{X_k\}$, WLLN holds good or not? 6
Prob. $\{ X_k = 1/\sqrt{k} \} = 2/3$ and Prob. $\{ X_k = -1/\sqrt{k} \} = 1/3$.

- Q2 a) Define hyper geometric distribution. Show that under certain conditions to be stated, hyper geometric distribution tends to binomial distribution. 9
b) Define a Beta distribution of second kind and find its mean and variance. 5

OR

- Q2 a) Define beta distribution of First kind. Obtain expression for its r^{th} raw moment. Hence derive its first four raw moments. 7
b) Define hyper geometric distribution. Obtain an expression for its mean and variance. 7
- Q3 a) For a bi-variate normal distribution, derive an expression for the conditional distribution of Y given $X = x$. 8
b) If 13 cards are chosen at random (WOR) from an ordinary pack of 52 cards. Find the probability that, 6
i) 6 – Cards are picture cards?
ii) None are picture card.

OR

Q3 a) Derive an expression for the marginal p. d. f. of Y for a bi-variate normal distribution. 9

b) For bi-variate normal distribution, show that X and Y are independent, iff $\rho = 0$. 5

Q4 a) i. State Lindberg-Levy's Central limit theorem. 6
ii. State Liapounoff's Central limit theorem.

b) Let $f(x) = 3x^2, 0 < x < 1$ 8
 $= 0$ otherwise

A random sample of size 15 is taken from above population.

Use CLT to find the estimated value of –

i. Prob. $\{ 0.6 < \bar{x} < 0.8 \}$

ii. prob. $\{ \bar{x} > 0.75 \}$

OR

Q4 a) State the Liapounoff's C.L.T. 4

b) Examine whether the Weak law of large number holds for the sequence $\{X_k\}$ of independent random variables defined as follows: 10

$$P[X_k = \pm 2^k] = 2^{-(2k+1)}, \quad P[X_k = 0] = 1 - 2^{-2k}.$$

Q5 a) Derive expression r^{th} raw moment of Beta distribution of Second kind its. Hence derive its Third and fourth raw moments. 8

b) Given the following bivariate normal distribution, 6

$$f(x, y) = k \cdot e^{-1/6[4(x+1)^2 - 2(x+1)(y-2) + (y-2)^2]}$$

Find the parameters of the distribution. Hence find k.

OR

Q5 a) Obtain an expression for the marginal p. d. f. of Y for a bi-variate normal distribution. 6

b) Obtain an expression for factorial moment generating function of hyper geometric distribution. 8