April-2016

Examination	:	B.Sc. Semester V
Subject	:	Statistics
Paper	:	ST 503 (Statistical Inference I)
Paper Code	:	4332
Total Marks	:	70
Duration of time	:	$2\frac{1}{2}$ Hours.

Instructions : (1) There are FIVE compulsory questions in this question-paper

(2) Statistical Tables will be provided upon request.

(3) Use of Scientific calculator is permitted.

Q1 (a) Define the following terms: 08

(1) Statistic (2) Estimates (3) Consistency

(4) Mean Square Error

(b) If $X_1, X_2 \ldots\ldots X_n$ is a random sample from an infinite population with mean μ and variance σ^2 then prove that (in usual notations) 06

$$V(\bar{X}) = \frac{\sigma^2}{n}$$

OR

Q1 (a) Define UMVUE . Prove that UMVUE, if exists , is unique. 09

(b) Show that (in usual notations) 05

$$MSE(T) = V(T) + [bias\ (T)]^2$$

Q2 (a) Prove that if T is an unbiased estimator of Θ and $\Psi(\Theta)$ is a linear parametric function of Θ then $\Psi(T)$ is an unbiased estimator of $\Psi(\Theta)$. 05

(b) Attempt the following : 09

(1) If e is the efficiency of a consistent estimator then show that $0 < e \leq 1$.

(2) If X_1, X_2, X_3 is a random sample from a p.d.f.

$$f((x,\theta) = \theta^x (1-\theta)^{1-x} \qquad x = 0, 1 \text{ and } 0<\Theta<1$$
$$= 0 \qquad \text{Elsewhere,}$$

Find the efficiency of $T = \frac{1}{7}(2X_1+4X_2+X_3)$ relative to the sample mean $\bar{X}$.

OR

Q2 (a) Let $X_1, X_2, \ldots\ldots X_n$ be a random sample from a p.d.f. $f(x, \Theta)$ where $\Theta \in \Omega$. Distinguish between the joint p.d.f. of $X_1, X_2, \ldots\ldots X_n$ and the likelihood function of any general observed sample $(x_1, x_2 \ldots\ldots x_n)$. 06

(b) Given $X_1, X_2, \ldots\ldots X_n$ is a random sample from 08

$$f(x, \theta) = \theta\, e^{-\theta x} \qquad 0<x<\infty\ , \quad \theta>0$$
$$= 0 \qquad \text{elsewhere}$$

Show that the sample mean $\bar{X}$ is an unbiased and consistent Estimator of $\frac{1}{\theta}$.

Q3 (a) State and prove the Cramer-Rao inequality applicable in case of a regular p.d.f. $f(x, \theta)$. 11

(b) If $X_1, X_2, \ldots\ldots X_n$ is a random sample from a Geometric Probability Distribution with p.d.f. 03

$$f(x, p) = (1-p)\, p^x \qquad x=0, 1, 2, \ldots\ldots\ldots \quad 0<p<1$$
$$= 0 \qquad \text{Elsewhere,}$$

Find the joint p.d.f. of $X_1, X_2, \ldots\ldots X_n$.

OR

Q3 (a) What is a MVB Estimator? Establish the necessary and sufficient condition for the existence of a MVB estimator for a parametric function $\varphi(\theta)$ in case of a regular density. 10

(b) Check whether a MVB estimator exists based on a random sample of size n from a Poisson probability distribution. 04

Q4 (a) Explain the logic of the Maximum Likelihood Method. Mention all properties of the Maximum Likelihood Estimators 10

(b) Estimate the mean θ of an exponential probability distribution based on a random sample of size 6 given below and using method of moments. 04

29.3 12.7 36.4 24.9 40.3 43.1

OR

Q4 (a) Explain the method of moments in detail. State its advantages and pitfalls. 10

(b) Estimate θ by the method of maximum likelihood based on a random sample of size n from 04

$$f(x,\theta) = (1-\theta)\,\theta^{x} \qquad x = 0,1,2\ldots\ldots\ldots, \quad 0<\theta<1$$
$$= 0 \qquad \text{Elsewhere}$$

Q5 (a) Write a note on Sufficiency , its usefulness and importance . 08

(b) Show that $\sum X_i$ is a sufficient statistic based on a random sample of size n from a Bernoulli probability distribution with mean p. $(0<p<1)$ 06

OR

Q5 (a) Estimate the parameter μ by the method of maximum likelihood if $X_1, X_2, \ldots\ldots X_n$ is a random sample from $N(\mu, 1)$. What is the M.L.E. of $2\mu + 7$? 10

(b) Define a Sufficient Statistic. State the Factorization theorem to find a single sufficient statistic. 04