Examination : B.Sc. Semester V
Subject : Statistics
Paper : ST 503 (Statistical Inference I)
Total Marks : 70
Duration of Time : $2\frac{1}{2}$ Hours

CODE NO :- 4332
MARCH - 2015

Q 1 (a) Define the following : 06

(1) Random Sample (2) Statistic (3) Consistent Estimator

(b) Given $X_1, X_2 \ldots\ldots X_n$ is a random sample from an infinite population with mean μ and variance σ^2. Show that (in usual notations) 08
(1) $E(\bar{X}) = \mu$ and (2) $E(S^2) = \sigma^2$.

OR

Q 1 (a) Prove that if T is an unbiased estimator of Θ and $\Psi(\Theta)$ is a linear parametric function of Θ then $\Psi(T)$ is an unbiased estimator of $\Psi(\Theta)$. 06

(b) Define UMVUE . Show that if it exists, it is unique. 08

Q 2 (a) Define (1) The most efficient estimator and (2) Relative Efficiency e of an estimator. Show that $0 < e \leq 1$. 09

(b) If X_1, X_2, X_3 is a random sample from a Poisson Probability Distribution with mean Θ , find the efficiency of $\frac{3X_1 + X_2 + 2X_3}{6}$ relative to $\bar{X}$. 05

OR

Q 2 (a) Establish the sufficient conditions of Consistency. 07

(b) If T_1 and T_2 are unbiased estimators of a parameter Θ with variances σ_1^2 and σ_2^2 respectively and with coefficient of correlation ρ , find the best linear unbiased combination of T_1 and T_2 . 07

Q 3 (a) State and Prove the Cramer-Rao inequality. Mention its uses. 09

(b) Find the Cramer-Rao lower bound for the variance of any unbiased estimator of (1) Θ (2) $e^{-\theta}$ based on a random sample of size n from 06

$$f(x,\Theta) = \Theta e^{-\Theta x} \quad x > 0 \quad \theta > 0$$
$$= 0 \quad \text{elsewhere}$$

OR

Q 3 (a) Define the Minimum Variance Bound (MVB) estimator of a parameter Θ. State and prove the Necessary and Sufficient condition for the existence of the MVB estimator of a parametric function in case of a regular p. d .f. 08

(b) Examine whether there exists the MVB estimator of any parametric function based on a random sample of size n from 06

$$f(x,\Theta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\theta)^2} \quad x \in R \quad \theta \in R$$

in case it exits, find its standard error.

Q 4 (a) Explain (1) Mean Square Error and Variance — 06

(2) The importance of the property of Sufficiency

(b) Let $X_1, X_2 \ldots\ldots X_n$ be a random sample from — 08

$$f(x,\Theta) = (1-\Theta)\,\Theta^x \qquad x = 0,1,2\ldots\ldots\ldots \qquad 0<\Theta<1$$

$$= 0 \qquad \text{elsewhere}$$

Show that the sample mean $\bar{X}$ is a sufficient statistic of Θ. Is it the Maximum Likelihood Estimator of Θ? Why?

OR

Q 4 (a) Explain the Method of Moments fully. State its advantages and drawbacks. — 08

(b) Let X follow an Exponential probability distribution with p. d. f. — 06

$$f(x,\Theta) = \frac{1}{\Theta} e^{-\frac{x}{\theta}} \qquad x > 0 \qquad \Theta > 0$$

$$= 0 \qquad \text{elsewhere}$$

Find the Maximum Likelihood Estimator of Θ.

Q 5 (a) Explain the Method of Maximum Likelihood and is logic by considering a random sample of size 3 from a Bernoulli Probability Distribution. — 08

(b) If $X_1, X_2 \ldots\ldots X_n$ is a random sample from a p. d. f. — 06

$$f(x,\Theta) = \frac{1}{\theta} \qquad 0<x<\Theta\Theta>0$$

$$= 0 \qquad \text{elsewhere}$$

Find the Maximum Likelihood Estimator of Θ.

OR

Q 5 (a) Examine whether — 07

$$f(x,\Theta) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2} \qquad -\infty<x<\infty\theta \in R$$

admits a sufficient statistic which is based on a r.s. of size n from it.

(b) State the properties of the Maximum Likelihood Estimators. — 07