

B.Sc. EXAMINATION:

SEMESTER-VI

CODE:4623

PAPER NO.:M-605

COMPLEX ANALYSIS-II

MARKS:70

TIME:2:30 HOURS

INSTRUCTIONS: (1) ALL QUESTIONS ARE COMPULSORY.  
(2) EACH QUESTION CARRY EQUAL MARKS.

- Q.1 A If complex function of complex variables is differentiable at a given point then it is continuous there, but converse is not true. 07
- B 1. Prove that  $f(z) = \sin hz$  is an entire function and also obtain  $f'(z)$ . 04
2. Show why function  $f(z) = xy + iy$  is nowhere analytic 03
- OR
- Q.1 A Obtain C-R condition for analytic function in polar form. 07
- B  $f(z) = z^3$  prove that  $f(z)$  is entire function, also obtain  $f'(z)$  and  $f''(z)$ . 07
- Q.2 A If  $f(z) = u + iv$  is analytic function then prove that  $u$  and  $v$  are both harmonic function. 07
- B Obtain Laplace's equation in polar form for analytic function  $f(z) = u + iv$  07
- OR
- Q.2 A Prove that  $u = 2x - x^3 + 3xy^2$  is harmonic function. Also obtain harmonic conjugate of  $u$  and corresponding analytic function. 07
- B If  $f(z)$  is analytic function of  $z$  then prove that, 07
- $$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] |\operatorname{Re} f(z)|^2 = 2 |f'(z)|^2$$
- Q.3 A If  $f(z)$  is analytic function in domain  $D$  and it is real valued function then it is constant. 07
- B  $f(z) = u + iv$  is analytic function of  $z = x + iy$  and  $\phi$  is function of  $x$  and  $y$  whose first and second order partial derivatives exists then 07
- prove that  $\left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 = \left[ \left( \frac{\partial \phi}{\partial u} \right)^2 + \left( \frac{\partial \phi}{\partial v} \right)^2 \right] |f'(z)|^2$
- OR
- Q.3 A Prove that  $u(r, \theta) = \log r, r > 0, 0 < \theta < 2\pi$  is harmonic function also obtain harmonic conjugate of  $u$ . 07
- B If  $f(z)$  satisfied C-R condition in polar form then prove that  $f'(z) = (\cos \theta - i \sin \theta) (u_r + iv_r)$ . 07
- Q.4 A If  $u$  and  $v$  both are harmonic functions of  $x$  and  $y$  and  $P = u_y + v_x$  and  $Q = u_x - v_y$  then prove that  $f(z) = P + iQ$  is an analytic function. 07
- B Find  $\operatorname{Res} (f(z), \text{singular point})$  for following functions: 07
1.  $f(z) = \frac{z+1}{z^2-2z}$  2.  $f(z) = \frac{\exp z}{z^2 + \pi^2}$

OR

Q.4	A	Find value of $\int_c \frac{z+1}{(z-2)(z-3)(z^2+2014)} dz$ where $c:  z  = 4$ .	07
	B	Find $\text{Res} (f(z), \text{singular point})$ for following functions: $f(z) = \frac{1}{z^3} \text{cosec} z^2$	07
Q.5	A	Discuss mapping $w = z^3$ .	07
	B	Discuss mapping $w = e^z$ and prove it is conformal.	07
	OR		
Q.5	A	State and prove cauchy residue theorem.	07
	B	Show that mapping $w = \frac{1}{z}$ transforms circles and lines into circles and lines.	07