B.Sc Sem. 6 Maths

: નોંધ : 27 OCT 2020 Sub. Code. 21858

૧. દરેક પ્રશ્નનો [a] અથવા [a(i)] અને [a(ii)] જ લખવાના રહેશે.

૨. પ્રશ્ન : ૧[a] અથવા ૧[a(i)] અને ૧[a(ii)] તથા ૨[a] અથવા ૨[a(i)] અને ૨[a(ii)] ના 14 માર્કસ ના બદલે ૧૮ માર્કસ રહેશે.

૩. પ્રશ્ન : ૩[a] અથવા ૩[a(i)] અને ૩[a(ii)] તથા ૪[a] અથવા ૪[a(i)] અને ૪[a(ii)] ના 14 માર્કસ ના બદલે ૧૭ માર્કસ રહેશે.

૪. દરેક પ્રશ્નનો પ્રશ્ન નં ૧(b), પ્રશ્ન નં ૨(b), પ્રશ્ન નં ૩(b) તથા પ્રશ્ન નં ૪(b) (ટુંકા પ્રશ્નો) વિદ્યાર્થીએ લખવાના નથી.

Q1 A Define ring and prove elementary properties of ring. Also prove that (Z, +, .) is a commutative ring. 14

OR

A(i) Prove that the set $R = \{a + b\sqrt{3} \,/ a, b \in \mathbb{R}\}$ is commutative ring under usual addition and multiplication. 07

A(ii) Prove that the intersection of two sub rings of a ring is a sub ring of a ring. 07

Q1 B Attempt any Three. 03

(i) True or False: In ring (R, +, .), (R, +) is commutative group.

(ii) Define: Characteristic of a ring.

(iii) Define: Commutative ring.

(iv) A ring $(\mathbb{R}, +, \cdot)$ is called ring with unity if......... (Fill in the blanks).

(v) Write Characteristic of a ring $(Z_6, +_6, \cdot_6)$

Q2 A Define field and prove that every finite integral domain is field. Also show that for a prime number p, $(Z_p, +_p, \cdot_p)$ is field. 14

OR

A(i) Prove that $(Z_7, +_7, \cdot_7)$ is an integral domain. 07

A(ii) $(\mathbb{C}, +, \cdot)$ and $(M_2(\mathbb{R}), +, \cdot)$ are rings and a function $\emptyset: \mathbb{C} \to M_2(\mathbb{R})$ is defined as $\emptyset(x + iy) = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$ then show that $\emptyset$ is a homomorphism. 07

Q2 B Attempt any Four. 04

(i) True or False: $(Z_4, +_4, \cdot_4)$ is an integral domain.

(ii) True or False: Every integral domain is a field.

(iii) Define: An integral domain

(iv) Which of the following ring is a field ....

(A) $(\mathbb{Z}, +, \cdot)$ (B) $(Z_8, +_8, \cdot_8)$ (C) $(Z_6, +_6, \cdot_6)$ (D) $(Z_5, +_5, \cdot_5)$

(v) Define: Isomorphism of rings.

(vi) Define: Ring without zero divisors.

Q 3 A If R is commutative ring with unity and an ideal M of R is maximal iff R/M is a field. 14

OR

A(i) Prove that field have no proper ideal. 07

A(ii) If $(\mathbb{Z}, +, \cdot)$ is a ring then prove that $I = 7\mathbb{Z}$ is ideal of ring $(\mathbb{Z}, +, \cdot)$. 07

Q3 B Attempt Any Three 03

(i) State Fermat's theorem.

(ii) Define: Improper ideal.

(iii) Define: Ideal.

(iv) Write any one ideal of ring $(Z_5, +_5, \cdot_5)$.

(v) True or False: Intersection of two ideals is also ideal of ring R.

Q4 A For an integral domain D prove that D[x] is also integral domain with respect to binary operation addition and multiplication of polynomials. 14

OR

A(i) Find sum and product of the polynomials $f = (2, 0, -8, 0, 4, 0, 0, \ldots\ldots)$ and $g = (1, -4, 0, 0, \ldots\ldots)$ 07

A(ii) State and prove remainder theorem for polynomial in F[x]. 07

Q4 B Attempt Any Four 04

(i) True or False: For the quaternion R^4, $\hat{k}^2 = -1$.

(ii) Define: Quaternion R^4

(iii) For $a \in \mathrm{R}^4$ and $a = a_1 + a_2\hat{\imath} + a_3\hat{\jmath} + a_4\hat{k}$ then $a^{-1} =$………. (Fill in the blank).

(iv) True or False: If f(x) = x +5 and g(x) = 2x + 7 are in Z_{11}[x] then f(x)+ g(x) = 3x + 1

(v) True or False: The degree of the polynomial is non- negative integer.

(vi) Define: Multiplication of two polynomials over ring.