B.Sc. Sem. - 6 Maths - 604 : નોંધ : 28 OCT 2020 Sub code 21859

૧. દરેક પ્રશ્નનો [a] અથવા [a(i)] અને [a(ii)] જ લખવાના રહેશે.

૨. પ્રશ્ન : ૧[a] અથવા ૧[a(i)] અને ૧[a(ii)] તથા ૨[a] અથવા ૨[a(i)] અને ૨[a(ii)] ના 14 માર્કસ ના બદલે ૧૮ માર્કસ રહેશે.

૩. પ્રશ્ન : ૩[a] અથવા ૩[a(i)] અને ૩[a(ii)] તથા ૪[a] અથવા ૪[a(i)] અને ૪[a(ii)] ના 14 માર્કસ ના બદલે ૧૭ માર્કસ રહેશે.

૪. દરેક પ્રશ્નનો પ્રશ્ન નં ૧(b), પ્રશ્ન નં ૨(b), પ્રશ્ન નં ૩(b) તથા પ્રશ્ન નં ૪(b) (ટુંકા પ્રશ્નો) વિદ્યાર્થીએ લખવાના નથી.

TOTAL MARKS: 70

Instruction: All questions are compulsory.

Q: 1 (A) Define: continuous function in metric space. Prove that continuous image of compact subset of metric space is compact. 14

OR

(A)(i) Prove that every Compact subset of metric space is closed. 07

(A)(ii) Prove that $\mathcal{R} - \{20\}$ is disconnected subset of $\mathcal{R}$ 07

Q: 1 (B) Attempt Any Three 03

(i) Which of the following is not true?

(A) union of two connected set is always connected

(B) Every Compact subset of $\mathcal{R}$ is bounded

(C) Every closed and bounded subset of $\mathcal{R}$ is compact

(D) there does not exist an interval which is not connected

(ii) Which of the following is Compact subset of $\mathcal{R}$?

(A) (1,2) (B) [1,2) (C) [1,2] (D) (1,2]

(iii) Which of the following is connected subset of $\mathcal{R}$?

(A) Set of all natural numbers (B) set of all integers

(C) Set of all rational numbers (D) union of rational and irrational numbers

(iv) Define: cover of a subset of metric space.

(v) Define: Compact subset of metric space.

Q: 2 (A) Let $X = \{1,2,3,4\}$ and 14

$\mathcal{T} = \{ X, \phi, \{1\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,4\}, \{1,2,4\}\}$

Prove that $(X, \mathcal{T})$ is topological space. for $A = \{1,3,4\}$ Find all interior point of A, limit point of A and closure of A.

OR

Q: 2 (A)(i) Prove that intersection of two topological spaces is topological space. 07

(A)(ii) $(X, \mathcal{T})$ is topological space and for $A \subset X$, $B \subset X$ 07

Prove (a) $A \subset B \Rightarrow int(A) \subset int(B)$

(b) $int(A \cap B) = int(A) \cap int(B)$

Q: 2 (B) Attempt any Four 04

(i) Define: Define comparable topological space.

(ii) Define: Define strong and Weak Topology.

(iii) True / False: Union of two topology is always topology.

(iv) Let X = {1,2,3} and topological space $\mathcal{T}_1$ = { X,ϕ, {2} } $\mathcal{T}_2$ = { X,ϕ, {3} } then $\mathcal{T}_1$ & $\mathcal{T}_2$ are ;

(A) Comparable topology (B) Non comparable topology
(C) Discrete topology (D) Indiscrete topology

(v) If X is any nonempty set and $\mathcal{T} = \{ \phi , X\}$ then $\mathcal{T}$ is .... topology

(vi) True / False; If (X, $\mathcal{T}$) is topological space and A, B ⊂ X then $A \subset B \Rightarrow \bar{A} \subset \bar{B}$

Q:3 (A) State and prove comparison test for convergence of improper integrals . 14

And Check convergence of $\int_1^{\infty} \frac{dx}{x\sqrt{x^2+1}}$

OR

Q:3 (A)

(i) State and prove Dirichlet's test for convergence of improper integral 07

(ii) Discuss convergence of $I = \int_0^{\infty} \frac{x^{2m}}{2 + x^{2n}} dx$ 07

Q:3 (B) Attempt any three. 03

(i) $I = \int_1^{\infty} \frac{1}{x}\, dx$ is Which kind of improper integral?

(ii) Discuss convergence of $I=\int_2^{3} \frac{1}{(x-3)}\, dx$

(iii) Discuss convergence of $I = \int_2^{\infty} x^{-2}\, dx$

(iv) Define: Absolute Convergence of improper integral.

(v) State: Abel's test for convergence of improper integral.

Q:4 (A) State and prove Weierstrass M-test for uniform convergence of sequence of function and discuss uniform convergence for 14

$<f_n(x)> = \frac{1}{n+x}$; $x \in [1, \infty)$, For $\forall n \in N$

OR

Q:4 (A)(i) Prove that : $\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|} \forall\, x, y \in \mathcal{R}$ 07

(ii) Prove that the set of rational number is not order complete. 07

Q:4 (B)(i) Attempt any four

True / False: Set of Natural numbers is countable 04

(ii) True / False: Union of two Countable set is Countable

(iii) Define: countable set

(iv) State: Weierstrass M-test for uniform convergence of series of function

(v) Discuss uniform convergence for $\sum_{n=1}^{\infty} \frac{\cos(nx)}{n^2+2n+1}$ $x \in \mathcal{R}, \forall n \in N$

(vi) In which interval $< fn(x) > = < x^n >$ is uniform convergence

(A) $x \in [0,1]$ (B) $x \in [0,1)$ (C) $x \in \mathcal{R}$ (D) $x \in [-1,0]$