B.Sc. Sem. 6 Sub Code: 21861

: નોંધ :

૧. દરેક પ્રશ્નનો [a] અથવા [a(i)] અને [a(ii)] જ લખવાના રહેશે.
૨. પ્રશ્ન : ૧[a] અથવા ૧[a(i)] અને ૧[a(ii)] તથા ૨[a] અથવા ૨[a(i)] અને ૨[a(ii)] ના 14 માર્કસ ના બદલે ૧૮ માર્કસ રહેશે.
૩. પ્રશ્ન : ૩[a] અથવા ૩[a(i)] અને ૩[a(ii)] તથા ૪[a] અથવા ૪[a(i)] અને ૪[a(ii)] ના 14 માર્કસ ના બદલે ૧૭ માર્કસ રહેશે.
૪. દરેક પ્રશ્નનો પ્રશ્ન નં ૧(b), પ્રશ્ન નં ૨(b), પ્રશ્ન નં ૩(b) તથા પ્રશ્ન નં ૪(b) (ટુંકા પ્રશ્નો) વિદ્યાર્થીએ લખવાના નથી.

Q-1 A Obtain Laplace equation in Cartesian form and in polar form for an analytic function f(z) = u + iv 14

OR

Q-1 A(i) If f(z) & $\overline{f(z)}$ are both analytic function in domain D ,then f is constant function in D. 07

A(ii) Find an analytic function f(z) = u + iv such that, u − v = x + y 07

Q-1 B Attempt any three. 03

(i) True or false: Ever an analytic function is an entire function.

(ii) An analytic function with constant modulo is …….(Fill the blank)

(iii) True or false: f(z) = $\bar{z}$ is an entire function.

(iv) Write C-R condition in polar form for an analytic function f(z)

(v) True or false: f(z) = sin z is entire function.

Q-2 A If f(z) = u + iv is analytic function of z = x + iy and 'w' is function of x and y and whose first and second order partial derivative exist ,then prove that 14

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=\left[\frac{\partial^2 w}{\partial u^2}+\frac{\partial^2 w}{\partial v^2}\right]\left|f'(z)\right|^2$$

OR

Q-2 A(i) Prove that u=x^2-y^2 & v=$\frac{y}{x^2+y^2}$ are both harmonic functions even though f(z) = u + iv is not analytic function. 07

A(ii) If v is harmonic conjugate of u and u is harmonic conjugate of v in some domain D then u and v are both constant in D. 07

Q-2 B Attempt any four. 04

(i) Define: Harmonic function.

(ii) True or false: u = x + y is a harmonic function.

(iii) True or false: If v is a harmonic conjugate of harmonic function u then f(z) = u + iv is an analytic function.

(iv) If u = $\frac{y}{x^2+y^2}$ then u_x = ……..(Fill the blank)

(v) If u = x^2-y^2 then u_{xx} = ……..(Fill the blank)

(vi) If u is harmonic function then $\frac{\partial^2 u}{\partial z \partial \bar{z}}$ = ……. (Fill the blank)

Q-3 A State and prove Cauchy integral formula and hence find $\int_{|Z|=4} \frac{1}{z(z^2-1)(z+2)}dz$ 14

OR

Q-3 A(i) State Cauchy integral formula and prove that $f^n(z_0)=\frac{n!}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^{n+1}}dz$ 07

A(ii) Evaluate $\int_C \pi e^{\pi\bar{z}}\,dz$, Where C is the square joining points z = 0, z = 1, z = 1 + I and z = i 07

Q-3 B Attempt any three. 03

(i) Find Res $\left(\frac{e^{2z}}{(z-1)^2},1\right)$

(ii) If C: $|z| = 2$ then $\int_c \frac{z}{z-1}\, dz =$ _______ (Fill in the blank)

(iii) Define: Residue of $f(z)$ at pole $z = z_0$

(iv) $\int_{|z|=1} \frac{dz}{z+5} =$ ……..(Fill in the blank)

(v) If C: $z \to z_0 = r_0 e^{i\theta}$ then $\int_c \frac{dz}{z-z_0} =$ _______ (Fill in the blank)

(vi) Define: singular point.

Q-4 A Obtain a transformation of sector $r \leq 1, 0 \leq \theta \leq \frac{\pi}{4}$ under the mapping $W = z^2$. Also prove that the transformation $W = 2z + z^2$ maps the unit circle $|z| = 1$ of Z-plane into a Cardiode in W-plane. 14

OR

Q-4 A(i) Show that the composition of two bilinear maps is again bilinear map. 07

A(ii) Show that mapping $w = \frac{1}{z}$ transforms circles and lines into circles and lines. 07

Q-4 B Attempt any four. 04

(i) True / False: $W = \bar{Z}$ is not conformal.

(ii) Write fixed points of the transformation $W = \frac{z-1}{z+1}$

(iii) Define : Mobious mapping.

(iv) Write inverse transformation of $W = \frac{z-i}{iz+1}$

(v) Define: conformal mapping.

(vi) The points which coincide with their transformation are called ……...(Fill the blank)