B. Sc Semester-VI Mathematics Examination March/April-2016
Linear Algebra - I
Paper No: M-302

Total Marks:70

Time : 2:30 hours
Paper code: 38024

Instructions:

(1) All questions are compulsory.
(2) Each question carry equal marks.

Que-1. (a) Let $\mathcal{P}_3(\mathbb{R})$ be the set of all polynomials of degree less than or equal to three. Prove that $\mathcal{P}_3(\mathbb{R})$ is a vector space over $\mathbb{R}$ with usual addition and scalar multiplication. [7]

(b) Define *linearly independent set.* Show that a subset of a linearly independent subset is always linearly independent. [7]

or

Que-1. (a) Let $V = \{(a,b) : a > 0, b > 0; a, b \in \mathbb{R}\}$. For $\alpha \in \mathbb{R}$ and $(a,b), (c,d) \in V$, $(a,b)+(c,d) = (ac, bd)$ and $\alpha(a,b) = (a^\alpha, b^\alpha)$. Show that V is a vector space over $\mathbb{R}$. [7]

(b) Show that the set $B = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ is linearly independent subset of $M_2(\mathbb{R})$. [7]

Que-2. (a) Show that $W = \{(x,y,z) : x + 2y - z = 0\}$ is a subspace of $\mathbb{R}^3$. Find its dimension. [7]

(b) Define *basis of a vector space.* In usual notation prove that any linearly independent set can be extended to basis. [7]

or

Que-2. (a) Let W_1 and W_2 be two subspaces of a vector space V. Prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$. [7]

(b) Give one basis of the vector space $\mathcal{P}_3(\mathbb{R})$ of all polynomials of degree less than or equal to 3 with real coefficients. Justify your answer. [7]

Que-3. (a) State and prove rank-nullity theorem. [7]

(b) Show that the transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined as $T(x,y,z) = (x+y+z, 2x-3y+4z)$ is linear. Find its kernel. [7]

or

Que-3. (a) Let U and V be two vector spaces and $T : U \to V$ be one one and onto linear transformation. Show that $T^{-1} : V \to U$ is also a linear transformation. [7]

(b) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by $T(x,y,z,t) = (x-y+z+t, x+2z-t, x+y+3z-3t)$. Obtain matrix representation of T relative to standard basis. [7]

Que-4. (a) Let U and V be two vector spaces and $T : U \to V$ be an isomorphism. Prove that $\dim(U) = \dim(V)$. [7]

(b) Consider the vector space $V = \mathcal{P}(\mathbb{R})$ of all polynomials over $\mathbb{R}$.
Let $D : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$ be defined by $D(p(x)) = \frac{dp(x)}{dx}$. Find nullity and rank of D. [7]

or

Que-4. (a) Show that a mapping $G : \mathbb{R}^3 \to \mathbb{R}^3$ defined as
$G(x, y, z) = (x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta, z)$ is linear. Find kernel of G. [7]

(b) Let U and V be two vector spaces having same dimensions. Show that $U \cong V$. [7]

Que-5. (a) Let $u = (x_1, x_2), v = (y_1, y_2) \in \mathbb{R}^2$. Define $\langle u, v\rangle = x_1y_1 - 3x_1y_2 - 3x_2y_1 + kx_2y_2$.
For which value(s) of k it will be an inner product on $\mathbb{R}^2$. Justify your answer. [7]

(b) Let V be an inner product space and $u, v \in V$. Prove that $|\langle u, v\rangle| = \|u\|\|v\|$
if and only if u and v are linearly dependent. [7]

or

Que-5. (a) State and prove triangle inequality. [7]

(b) Show that each of the following is not an inner product on $\mathbb{R}^3$.
For $u = (x_1, x_2, x_3)$ and $v = (y_1, y_2, y_3)$,
(i) $\langle u, v\rangle = x_1y_1 + x_2y_2$
(ii) $\langle u, v\rangle = x_1y_2x_3 + y_1x_2y_3$. [7]