20 SEP 2019

B.Sc. SEM VI

PAPER NO.: MAT-CC- 606

COMPLEX ANALYSIS – II

TIME: 02 :30 HOURS | CODE NO: 21861 | TOTAL MARKS:70

INSTRUCTIONS: (1) ALL QUESTIONS ARE COMPULSORY.
(2) EACH QUESTION CARRY EQUAL MARKS

Q.1 A Obtain C-R condition for an analytic function in Cartesian form. 7

B If f(Z) and $f(\bar{Z})$ are both analytic functions in domain D then prove that function f is constant in domain D. 7

OR

Q.1 A If f(Z) is an analytic function of Z then prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$ 7

B Obtain C-R condition for an analytic function in Polar form. 7

Q.2 A Obtain Laplace equation in Cartesian form for an analytic function f(Z). 7

B If f(z) = u + iv is an analytic function of z and ψ is function of x and y and first order and second order partial derivatives of ψ exists then prove that 7

$\psi_{xx} + \psi_{yy} = [\psi_{uu} + \psi_{vv}]\,|f'(z)|^2$

OR

Q.2 A If V is harmonic conjugate of U and U is harmonic conjugate of V in some domain D then prove that U and V are both constant in D. 7

B If f(Z) is an analytic functions in domain D then prove that 7

$(\frac{\partial}{\partial x}|f(z)|)^2 + (\frac{\partial}{\partial y}|f(z)|)^2 = |f'(z)|^2$

Q.3 A If f(z) is an analytic inside and on the boundary C then prove that 7

$f'(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^2}dz$

B Using Cauchy integral formula evaluate $\oint_C \frac{2z+3}{(z^2-4)(z+3)}dz.$ where C is $|z| = 4$ 7

OR

Q.3 A Evaluate : (1) $\int_o^{1+i}(\bar{z})^2$ dz along 2y = x (2) $\int_o^{1+i}(x^2 + ix^2)$ dz along y = x +1 7

B Prove that if f(z) is an integrable along a curve C having finite length L and if there exist a positive real number M such that $|(f(z)| \leq M$ then $|\oint_C f(z)\,dz| \leq ML$ 7

Q.4 A Using Cauchy's residue theorem, solve: $\int_C \frac{1-2z}{z(z-1)(z-2)}$ dz Where C: $|Z| = 3$ 7

B Discuss the method to find residue of function f(z) at pole Z_0 of order m. 7

OR

Q.4 A State and prove Cauchy's residue theorem. 7

B Using Cauchy's residue theorem, solve: $\int_C \frac{1}{Z^4-1}$ dz Where C: $|Z| = 2$ 7

Q.5 A Discuss Mapping $W = Z^2$ in Cartesian form and in polar form. 7

B Prove : (1) $W = \bar{Z}$ is not conformal. 7

(2) Obtain transformation angle of mapping $W = \frac{1}{Z}$ at points z = 1 and z = i

OR

Q.5 A Discuss Mapping $W = \frac{1}{Z}$ in Cartesian form and in polar form. 7

B Prove : (1) $W = Z^2$ is conformal. 7

(2) Obtain transformation angle of mapping $W = \frac{1}{Z}$ at points z = 1 and z = i