17 SEP 2019

B.Sc. SEM VI EXAMINATION:

PAPER NO.: MAT-CC- 603

RING THEORY

CODE NO: 21858

TOTAL MARKS:70

TIME: 02 :30 HOURS

INSTRUCTIONS: (1) ALL QUESTIONS ARE COMPULSORY.

(2) EACH QUESTION CARRY EQUAL MARKS

Q.1	A	$(Z ,+ , \cdot)$ is a ring and $S = \{7m / m \in Z\}$. Prove that S is a sub ring of a ring $(Z ,+ , \cdot)$	07
	B	$(R ,+ , \cdot)$ is a commutative ring with unity. If for $a,b \in R$, $a \oplus b = a + b + 1$ and $a \odot b = a + b + ab$ then prove that $(R ,\oplus , \odot)$ is ring.	07
		OR	
Q.1	A	State and prove elementary properties of ring.	07
	B	Prove that intersection of two sub rings of ring R is a sub ring of a ring R.	07
Q.2	A	Prove: Finite integral domain D is a field.	07
	B	Prove; Intersection of two ideals of ring R is an integral domain.	07
		OR	
Q.2	A	Prove: Every filed an integral domain but converse is not true.	07
	B	Prove: Field has no proper ideal.	07
Q.3	A	Using Euler's theorem, find remainder when 3^{256} is divisible by 4.	07
	B	State and prove Fermat's theorem.	07
		OR	
Q.3	A	Using Fermat's theorem, find remainder when 3^{51} is divisible by 7.	07
	B	Find all maximal ideals and principal ideals of $(Z_{12} , +_{12} , \cdot_{12})$.	07
Q.4	A	If f = (1, 2, 4, 0 ,0 4 ,0,0,......) and g = (2,1,6,4,0,5,0,0,....) are two polynomials in ring $Z_7[x]$ then find f + g and f.g.	07
	B	For non zero polynomials f(x), g(x) $\in$ F[x] prove that deg(f(x)g(x)) = deg(f(x)) + deg(g(x))	07
		OR	
Q.4	A	Find G.C.D of f = (1, 2, 3, 5 ,0,0,......) and g = (2,1,3,4,0,5,0,0,....) in $Z_7[x]$ and express it into form a(x)f(x) + b(x)g(x).	07
	B	If f = (1, -2, 0, 3 ,0,0,......) and g = (2,0,6,-3,0,4,0,0,....) are two polynomials in ring Z [x] then find f + g and f.g.	07
Q.5	A	State and prove factor theorem for ring of polynomials.	07
	B	State Eisenstein criterion for irreducibility of polynomials in F[x] and hence prove that $2x^{10} - 25x^3 + 10x^2 - 30$ is irreducible over Q[x]	07
		OR	
Q.5	A	State and prove division algorithm for ring of polynomials.	07
	B	Prove that $x^4 + 3x^3 + 2x + 4$ is reducible over $Z_5[x]$ and obtains its all factors.	07