B.Sc. EXAMINATION:- 2015 March

SEMESTER-VI

RING THEORY

PAPER NO.:M-601

TIME:2:30 HOURS

CODE NO:4619

TOTAL MARKS:70

INSTRUCTIONS(1)ALL QUESTIONS ARE COMPULSORY.
(2)EACH QUESTION CARRY EQUAL MARKS.

Q.1 A If R is Boolean ring then prove that (1)for a∈ R; a+a = 0 (2)For a,b ∈ R; a+b = 0 ⇒ a=b (3) R is commutative ring. (4) find characteristics of R. 07

B If R is Boolean ring and $a^3 = a$ then prove that R is commutative ring. 07

OR

Q.1 A If $M_2\ (F)$ is the set of matrices over field F then show that ($M_2(F)$, + , •) is ring with unity. 07

B Define sub ring ,State and prove necessary and sufficient condition for nonempty subset S of ring R to be a sub ring of R. 07

Q.2 A If $R \times Z$ is the set of all ordered pair (a ,b) is define as (a , b) $\oplus$ (c , d) = (a $\oplus$ c , b $\oplus$ d) and (a , b) $\odot$ (c ,d) = [$a_{\odot}$ c + $b_{\odot}$ c + $d_{\odot}$ a] then show that ($R \times Z$, $\oplus$, $\odot$) is ring. 07

B If f is homomorphism of ring R into ring R' with kernel S then S is an ideal of R. 07

OR

Q.2 A Every finite integral domain is a field. 07

B Prove that field has no proper ideal. 07

Q.3 A State and prove fundamental theorem on homomorphism of ring. 07

B Φ: (R ,+ ,•)→(R′,⊕,⊙) is homomorphism 07

(1) If u is sub ring of R then ϕ(u) is sub ring of R′

(2) If I is ideal of R then ϕ(I) is ideal of R′.

OR

Q.3 A If R is commutative ring , a ∈ R and I = { $ax = 0$ / x ∈ R } then I is an ideal of R. 07

B Show that I is an idal of I+M where I is any ideal of ring R, and M is any sub ring of R. 07

Q.4 A Define UFD and prove that P is prime in PID iff <p> is maximal ideal in D. 07

B If a is non-zero non unit element of PID then the factorization is unique. 07

OR

Q.4 A State and prove factor theorem. 07

B Give an example of sub ring of ring M_2 which is right ideal but not left ideal. 07

Q.5 A If $f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$ and $g(x) = x^2 - 2x + 3 \in Z_5[X]$ then find q(x) and r(x) such that f(x) = q(x)·g(x) + r(x). 07

B If D is an integral domain then the polynomial ring D[x] is also an integral domain 07

OR

Q.5 A f = (0 , 1 , 2 , 0 , 0 ......) ; g = (1 , 0 , −3 , 1 , 0 ....) f, g ∈ Z[X] then obtain $f + g$ and $f \cdot g$ 07

B State and prove Euler theorem. 07