

ST -603 [Sampling Theory]

Time: $2\frac{1}{2}$ Hours

Marks: 70

Instructions:

1. There are FIVE compulsory Questions in this paper. Each question carries 14 marks.
2. Statistical tables & Graph Papers will be provided on request.
3. Use of scientific calculator is allowed.

Q 1. a) In Simple random sampling (WOR) for proportions, prove that (in usual notations) 8

i. $E(p) = P$

ii. $V(p) = \frac{N-n}{N-1} \frac{P(1-P)}{n}$

b) In a simple random sample of 200 from a population of 1600 colleges, 120 colleges were in favour of a proposal. 6

Estimate the total number of colleges in the population that favoured the proposal and also the standard error of the estimator used.

OR

Q. 1. a) With respect to SRSWOR for proportions, by defining a characteristic variable y on each population unit, show that (in usual notations) 10

i. $\bar{y} = p$

ii. $V(\hat{A}) = \left(\frac{N-n}{N-1}\right) \frac{N^2 PQ}{n}$

b) A random sample of 100 items is taken from a normal population of size 1200. It is known that a sample contains 5% defective items then find 95% Confidence interval for the proportion of the defectives items. 4

Q 2. a) Determine sample size for estimating population proportion, when margin of error, confidence coefficient are given, in case of SRSWOR from a finite population. 9

b) There are 1500 sixth graders in a city. To estimate the mean score (\bar{Y}) they would get if they took a certain achievement test, find the sample size required if it is desired to estimate \bar{Y} with a margin of error 3 with a confidence coefficient 0.90. the population standard deviation S is considered to be 9.4. 5

OR

Q 2. a) Determine sample size for estimating population mean when margin of error and confidence coefficient is given, in case of SRSWOR. 8

- b) A population of N units is divided into two strata with $N_1:N_2 = 2:3$ and $S_1:S_2 = 3:2$ in usual notations determine the ratio $n_1:n_2$ for the optimum allocation for fixed sample size if $n=50$, find the value of n_1 & n_2 . 6

- Q 3. a) In case of stratified random sampling, prove that (in usual notations) 9

$$(1) E(\bar{y}_{st}) = \bar{Y}$$

$$(2) V(\bar{y}_{st}) = \frac{1}{N^2} \sum_{h=1}^L N_h(N_h - n_h) \frac{S_h^2}{n_h}$$

$$(3) V(\bar{y}_{st})_{prop} = \frac{1-f}{n} \sum_{h=1}^L W_h S_h^2$$

- b) An investigator wishes to take a stratified random sample of size 80 with following information regarding the population 5

Stratum	N_h	S_h
1	400	10
2	600	20

Allocate the total stratified random sample size to these strata under

- (1) Proportional Allocation and
(2) Neyman Allocation.

OR

- Q 3. a) Derive formula of sample size for proportional and optimum allocations in stratified random sampling. 8

- b) Explain real life situation where Stratified Random Sampling is appropriate. 6

- Q 4. a) Prove that, for syastematic sampling, 10

$$V(\bar{y}_{sy}) = \frac{N-1}{N} S^2 - \frac{k(n-1)}{N} S_{wsy}^2 \text{ (in usual notations)}$$

- b) What is stratification? Explain the criteria of stratification. 4

OR

- Q 4. a) Prove that if the term in $\frac{1}{N_h}$ are ignored, 10

$$V_{opt.} \leq V_{pro.} \leq V_{ran.} \text{ (in usual notations)}$$

- b) What is cluster sampling. Write merits and demerits of cluster sampling. 4

- Q 5. a) Prove that, (in usual notations) 9

$$V(\bar{y}_{sy}) = \frac{S^2}{n} \frac{N-1}{N} [1 + (n-1) \rho_w]$$

- b) $V(\bar{y}_{st})$ is minimum for fixed total size of sample (n) if $n_h \propto N_h S_h$ 5

OR

- Q 5. a) Explain the advantages and disadvantages of Systematic Sampling. 7

- b) Show that systematic sampling is more precise than SRS if and only if 7

$$S_{wsy}^2 > S^2$$