

Statistics Paper ST 603

(Sampling Theory) - 21627

Total Marks : 70

Duration of Time : $2\frac{1}{2}$ Hours.

Instructions: There are Five compulsory questions in this question-paper. All questions carry equal marks. Statistical Tables will be provided upon request.

- Q 1 (a) In simple random sampling without replacement for proportions, by defining a variable characteristic y assuming values either 0 or 1 prove that (in usual notations) 10

$$(1) \bar{Y} = P$$

$$(2) S^2 = \frac{NPQ}{N-1}$$

- (b) In a simple random sample of 60 students selected without replacement from a college having enrolment of 750 students, 23 students were staying in hostels. Estimate the total number of students of this college staying in hostels and also estimate the standard error of the estimator you used. 04

OR

- Q 1 (a) With reference to simple random sampling without replacement for proportions, show that (in usual notations) 10

$$(1) E(p) = P$$

$$(2) V(p) = \frac{N-n}{N-1} \frac{PQ}{n}$$

- (b) Explain the procedure of constructing an asymptotic 95% confidence interval for population proportion P of units possessing a certain characteristic in the population. 04

- Q 2 (a) Describe the procedure to estimate the sample size in case of SRSWOR from a finite population with units possessing a variable characteristic y and when the margin of error d in the estimation of the population mean as well as the confidence coefficient are specified. 08

- (b) Estimate the sample size required to estimate the mean weight of 2500 candies with the permissible margin of error 0.07 grams and the confidence coefficient 0.95 provided a preliminary estimate of the population standard deviation is 0.3 grams. 05

OR

Q 2 (a) Explain the following terms:

- (1) Sampling Error
- (2) Non sampling Error
- (3) Cluster Sampling

08

(b) What is a Stratified Random Sampling? Explain its advantages.

06

Q 3 (a) In Stratified Random Sampling, prove that (in usual notations)

09

$$(1) E(\bar{y}_{st}) = \bar{Y}$$

$$(2) V(\bar{y}_{st}) = \frac{1}{N^2} \sum_{h=1}^L N_h (N_h - n_h) \frac{S_h^2}{n_h}$$

(b) Allocate a Stratified random sample of total size 60 to be taken without replacement to three strata of the population with the Proportional allocation using the information given below in usual notations. Further find the $V(\bar{y}_{st})_{prop}$.

05

| Stratum No. | N_h | S_h |
|-------------|-------|-------|
| 1 | 300 | 10.1 |
| 2 | 600 | 13.2 |
| 3 | 100 | 5.7 |

OR

Q 3 (a) Explain the Neyman (optimum) allocation for the fixed total sample size n of a Stratified random sample. Derive $V(\bar{y}_{st})_{opt}$ and also mention its unbiased estimator

10

(b) Explain the procedure to draw a systematic random sample when Population size $N=50$ and the sample size $n=5$. Point out the operational advantage of systematic sampling compared to simple random sampling.

05

Q 4 (a) In usual notations prove that $V(\bar{y}_{st})_{opt} \leq V(\bar{y}_{st})_{prop} \leq V(\bar{y})_{ran}$ provided the fractions $\frac{1}{N_h}$ ($h=1,2,\dots,L$) can be ignored.

10

(b) Explain the applications of Cluster Sampling.

04

OR

- Q 4 (a) Based on the information given below (in usual notations), estimate the population mean \bar{Y} and also the standard error of your Estimator of the population mean. 07

| Stratum No. | N_h | observed y_{hi} values |
|-------------|-------|-------------------------------|
| 1 | 60 | 3.3, 3.9, 4.7, 3.1, 5.1, 4.2. |
| 2 | 40 | 9.5, 7.8, 8.7, 9.3. |

- (b) Obtain the expression of $V(\bar{y}_{sy})$ which involves S_{wsy}^2 and S^2 . 07

- Q 5 (a) If ρ_w is the coefficient of correlation between the units of the same systematic sample, then prove that (in usual notations) 09

$$V\left(\bar{y}_{sy}\right)=\left(\frac{N-1}{N}\right)\left(\frac{S^2}{n}\right)\left(1+(n-1) \rho_w\right)$$

- (b) Show that a systematic sample can be considered as drawing one unit from each of n strata in which population can be thought to be classified when $N=nk$ (in usual notations) 05

OR

- Q 5 (a) Describe the procedure to estimate the sample size when the aim is to predict the Population proportion P with the specified margin of error along with specified confidence coefficient. 09
- (b) Explain the gain due to stratification over the simple random sampling. 05