M.Sc. Physics Examination

Semester – I

Paper No – C103 Mathematical Methods in Physics

Paper Code – 4514

Time: 2 Hours 30 min march - 2017 Maximum Marks 70

Notes: (1) - All questions are compulsory. (2) Number in square bracket indicate marks

Que–1:

(a) Calculate the divergence of the vector field $\vec{r}/r^3$ using all the three coordinate systems. **[9]**

(b) Obtain scale factors in case of spherical polar coordinate system. [5]

OR

(a) Calculate divergence for position vector using (i) Cartesian (ii) Spherical polar and (iii) Cylindrical coordinate systems. **[9]**

(b) Prove that curl of electric field is zero. **[2]**

(c) Show that $-i\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)=-i\frac{\partial}{\partial \varphi}$ [3]

Que–2:

(a) Find the value of the integral

$\oint_C \frac{z^3 dz}{z^2-5z+6}$ Where C is a closed contour defined by the equation $2|z|-5=0$

[7]

(b) Evaluate $\oint \frac{\sin 2z}{6z-\pi} dz$ where circle has radius equal to 3. **[5]**

(c) Find the residues of the following function at given point **[2]**

$$\frac{\sin z}{(1-z^4)} at\ z=i$$

OR

(a) Evaluate the following integral using residue theorem **[8]**

$$\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$$

(b) Write Cauchy-Riemann conditions in Cartesian and Polar form. And show that z and z* are analytic or not. **[6]**

Que–3:

(a) Show that the Bessel's Functions are orthogonal. **[7]**

(b) Solve the differential equation $\frac{d^2y}{dx^2} + cotx\frac{dy}{dx} + 4y\,cosec^2x = 0$, by changing the independent variable method. **[4]**

(c) Show that Bessel's Function $J_n(x)$ is an even function when n is even and is odd function when n is odd. **[3]**

OR

(a) Solve given differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$ [7]

(b) Prove the recurrence formula for Bessel' function

$2J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$. **[4]**

(c) Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$. **[3]**

Que–4:

(a) Obtain the generating function for Legendre's Polynomials. **[7]**

(b) Find the Laplace transform of $t^2 sint$. **[4]**

(c) prove that $\int_{-1}^{+1} Pm(x)Pn(x) = 0$ as orthogonality of Legendre polynomial [3]

OR

(a) Show the recurrence formulae for Legendre's Polynomials

$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$ And

$(x^2-1)P'_n(x) = n[xP_n(x) - P_{n-1}(x)]$. **[7]**

(b) Find the Laplace transform of $(1+\sin 2t)$. **[4]**

(c) Write the equation for Hermite and Lagurerre's Polynomial. **[3]**

Que–5:

(a) Evaluate $\int_0^{\infty} \frac{e^{-at} - e^{-bt}}{t} dt$. **[6]**

(b) Find the Fourier Transform of te^{-at} . **[2]**

(c) Explain the uses of Laplace and Fourier transform citing examples. **[6]**

OR

(a) Using Laplace transforms, find the solution of the initial value problem (IVP)

$y'' + 9y = 6\cos 3t$, $y(0) = 2$, $y'(0) = 0$.

[5]

(b) Find the inverse Laplace transform of $\frac{s^2+3}{s(s^2+9)}$. **[3]**

(c) Using method of Fourier series, decompose the Triangle wave into sine and cosine forms. **[6]**