M.Sc. (Mathematics) Semester – 1

Paper – 3: Differential Equations (Code: 2750)

Time: 2 ½ Hours APRIL-2016 Total Marks: 70

Q.1 (a) Find eigenvalues and corresponding eigenfunctions for the equation $y'' + \lambda y = 0$ in each of the following cases: [7]

(i) $y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 0$ (ii) $y(0) = 0,\ y(2\pi) = 0$

(b) Classify the following p.d.e.: [7]

(i) $4r + 3s - 2t = 0$ (ii) $3\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} + 6\frac{\partial z}{\partial x} + 7\frac{\partial z}{\partial y} = 0$

OR

Q.1 (a) State Picard's iteration method. Use it to find first three successive approximation of $\frac{dy}{dx} = x + y,\ y(0) = 1.$ [7]

(b) (i) Show that $f(x,y) = xy^2$ satisfies the Lipschitz's condition on the rectangle $|x| \le 1,\ |y| \le 1$ and find the corresponding Lipschitz's constant. [7]

(ii) Prove that $f(x,y) = x^2cos^2y + ysin^2x$ satisfies the Lipschitz's condition.

Q.2 (a) Show that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0(x).$ [7]

(b) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\, sinx.$ [7]

OR

Q.2 (a) Prove that $\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1}(x)$ [7]

(b) Prove that $\frac{d}{dx}\left(x^{-n} J_n(x)\right) = -x^{-n} J_{n+1}(x)$ [7]

Q.3 (a) Solve by Series method: $y'' - xy = 0.$ [7]

(b) State and prove Rodrigue's formula. [7]

OR

Q.3 (a) In usual notation prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$, if $m \neq n.$ [7]

(b) Express the following function in terms of Legendre's Polynomials: [7]

$$f(x) = 4x^3 - 2x^2 - 3x + 8$$

Q.4 (a) Solve by direct integration: [7]

$\frac{\partial^2 z}{\partial x^2} = a^2 z$, given that when $x = 0, \frac{\partial z}{\partial x} = a\sin y$ and $\frac{\partial z}{\partial y} = 0$.

(b) State one dimensional wave equation and solve it by direct integration [7]

OR

Q.4 (a) Solve: $(2xy + z^2)dx + (x^2 + 2yz)dy + (y^2 + 2xz)dz = 0$. [7]

(b) Describe method to solve $Pp + Qq = R$. [7]

Q.5 (a) Solve: $(y^2 + yz + z^2)dx + (z^2 + zx + x^2)dy + (x^2 + xy + y^2)dz = 0$ [7]

(b) Show that $xp = yq$ and $z(px + yq) = 2xy$ are compatible and solve them. [7]

OR

Q.5 (a) Solve: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$ [7]

(b) Solve: $(D^3 - 4D^2D' + 4DD'^2)z = 24xy$. [7]
