November -2015

M.sc Mathematics Semester – I C.No : 2750

Paper No : 03 Differential Equations Marks : 70

Q.1 (a) Solve $y''+y=0$ by power series method. [7]

(b) State Bessel's differential equations and prove that $J_{\frac{1}{2}}(x)=J_{-\frac{1}{2}}(x)\tan x$ [7]

OR

Q.1 (a) Prove that $J_4(x)=\left(\frac{48}{x^3}-\frac{8}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x)$ [7]

(b) Express $f(x)=x^4+3x^3-x^2+5x-2$ in terms of Legendre's polynomial . [7]

Q.2 (a) State and prove Rodrigue's formulae. [7]

(b) Prove that $\int_{-1}^{1} p_m(x)p_n(x)dx=0$ if $m\neq n$. [7]

OR

Q.2 (a) Prove that $\frac{d}{dx}\left(x^{-n}J_n(x)\right)=x^nJ_{n+1}(x)$ [7]

(b) Prove that $J_{n-1}(x)+J_{n+1}(x)=\frac{2n}{x}J_n(x)$ [7]

Q. 3 (a) Solve $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ by method of separation of variable.
[7]

(b) Solve $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ given that $u=3e^{-y}-e^{-5y}$ *when* $x=0$. [7]

OR

Q.3 (a) A tightly stretched string with fired ends $x=0$ and $x=l$ is initially in a position $y(x,0)=y_0\sin^3\left(\frac{\pi x}{l}\right)$. If it released at rest from this position , find the displacement $y(x,t)$.[14]

Q. 4 (a) Discuss with all details Lagrange's method to solve *P.D.E.* $Pp+Qq=R$ [7]

(b) Solve $dx+dy+(x+y+z+1)dz=0$ by Natani's method. [7]

OR

Q.4 (a) Form the partial differential equations for : (i) $z=(x+y)f(x^2-y^2)$ [7]

(ii) $F(x+y+z,xyz)=0$

(b) Solve $(yz+2x)dx+(zx+2y)dy+(xy+2z)dz=0$ [7]

Q.5 (a) Find first three approximation of I.V.P. $\frac{dy}{dx}=2xy$, $y(0)=1$ by Picard's method [7]

(b) Solve by direct integration (i) $\frac{\partial^2 z}{\partial x^2}=a^2 z$ [7]

(ii) $\frac{\partial^2 z}{\partial x \partial y}=\cos(3x+2y)$

OR

Q.5 (a) Solve : $(2D^2-5DD'+2D'^2)z=5\sin(2x+y)$
[7]

(b) Solve : $(D^3-6D^2D'+11DD'^2-6D'^3)z=e^{5x+6y}$ [7]