Code: 2751   M.Sc. (Mathematics) Semester–1   Seat No. _____

Paper – 4: Number Theory (Old)

Time: 1 ½ Hours]   2 3 FEB 2021   [Total Marks: 42

Attempt any three questions out of four.

Q.1   Explain: Euclid's algorithm for determination of gcd of two integers.   [14]

OR

Q.1 (i)   Prove that if $ca \equiv cb \pmod{n}$ then $a \equiv b \pmod{\frac{n}{d}}$, where   [7]
$d = \gcd(c, n)$.

(ii)   Find the remainder when the integer $1^5 + 2^5 + \cdots + 51^5$ is   [7]
divided by 3.

Q.2   Prove or disprove: "Every positive integers can be expressed as   [14]
product of prime; this representation is unique apart from the
order in which the factors occur".

OR

Q.2 (i)   Show that: The number $\sqrt{2}$ is irrational.   [7]

(ii)   Show that there are infinitely many primes of the form $4k + 1$.   [7]

Q.3   Prove or disprove: Converse of Wilson's theorem.   [14]

OR

Q.3 (i)   Prove that the integer $111^{333} + 333^{111}$ is divisible by 7.   [7]

(ii)   Solve: $x \equiv 1 \pmod{3}, x \equiv 2 \pmod{5}, x \equiv 3 \pmod{7}$ [7]

Q.4   Prove that the Euler's totient function is multiplicative.   [14]

OR

Q.4 (i)   Find the formula of $\tau(n)$ for $n > 1$.   [7]

(ii)   Prove that: σ is multiplicative.   [7]