M. Sc. Semester-I (MATHEMATICS)

Paper 2: Topology-I

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Marks:70

Q-1. (a) Define a topological space. Let $\tau = \{A \subseteq R / A \text{ is uncounatable or } A = \phi\}$. Is τ a topology on R? Justify your answer. [06]

(b) For any subsets A and B of a topological space X, prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$ and $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$. [08]

OR

Q-1. (a) Define Base and Subbase for a topological space. Give examplesof each. [07]

(b) Is the set of rational numbers Q open in R? Justify your answer. [07]

Q-2. (a) Let A be a subset of a topological space X and let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$. [07]

(b) Prove that every subspace of a T_2-space X is a T_2-space. [07]

OR

Q-2. (a) Let A be a subset of a T_1-space X. Prove that A' is closed in X. [07]

(b) Prove that every metric space is a T_2-space. [07]

Q-3. (a) Let X and Y be topological spaces and $f : X \to Y$ be a continuous map. Prove that $f(\overline{A}) \subset \overline{f(A)}$ for every subset A of X. [07]

(b) Prove that every closed subspace of a complete metric space is complete. [07]

OR

Q-3. (a) State and prove Cantor's Intersection Theorem . [14]

Q-4. (a) Prove that the continuous image of a connected space is connected. [07]

(b) Let A be a connected subset of a topological space X and B be any subset of X with $A \subset B \subset \overline{A}$. Prove that B is connected. [07]

OR

Q-4. (a) Define: path connected space and locally connected space. [06]

(b) Let X and Y be connected subspaces. Prove that $X \times Y$ is connected. [08]

Q-5. (a) Prove that every compact metric space is bounded. [06]

(b) Prove that every continuous function f from a compact metric space X into a metric space Y is always uniformly continuous. [08]

OR

Q-5. (a) Prove that every compact metric space is complete. [07]

(b) Prove that compactness is a topological property. [07]

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