Nov- 2015

M.sc Mathematics Semester – I C.No : 2749

Paper No : 2 Topology – I Marks : 70

Q.1 (a) Write topological space. Construct three topological τ_1, τ_2, τ_3 such that $\tau_1 \subset \tau_2 \subset \tau_3$. [7]

(b) In usual notations prove the following results. [7]

(i) $\overline{\phi} = \phi$

(ii) $A \subset B \Rightarrow \overline{A} \subset \overline{B}$

(iii) $\overline{\overline{A}} = \overline{A}$

OR

Q.1 (a) Define base for a topology. Let X,Y,Z be topological spaces. And $f: X \to Y$, $g: Y \to Z$ be continuous then prove that $gof: X \to Z$ is continuous. [7]

(b) In a topological space (X,τ) Prove that an arbitary intersection of closed set is closed. And finite union of closed sets is closed. [7]

Q.2 (a) Let X and Y be topological spaces. Prove that a map $f: X \to Y$ is continuous if and only if The inverse image under f of every closed set in Y is closed in X. [7]

(b) State and prove pasting lemma. 7]

OR

Q.2 (a) In usual notation prove that $\overline{A} = A \cup A'$. [7]

(b) State and prove uniform limit theorem. [7]

Q. 3 (a) Prove that limit of a sequence in a metric space is unique [7]

(b) Prove that limit of a sequence in a T_2 space is unique. 7]

OR

Q.3 (a) Prove that contraction on X is a unique. [7]

(b) Prove that Z is nowhere dense set. [7]

Q. 4 (a) Prove that connectedness is a topological property. [7]

(b) State and prove intermediate value Theorem. [7]

OR

Q.4 (a) If $X \times Y$ is connected space then prove that X and Y are connected. [7]

(b) Prove that every path connected space is connected. [7]

Q. 5 (a) Prove that every finite topological space is compact. [7]

(b) Prove that every indiscrete topological space is compact. [7]

OR

Q.5 (a) Every compact metric space is bounded. [7]

(b) Prove that compactness is a topological property. [7]