17 OCT 2019

M.Sc. Physics Semester- 1 examination
PHYS-C-103: {Mathematical Methods in Physics}
Paper Code: 4514

Time: 2 Hours 30 Min **Maximum Marks: 70**

Note: Answer all questions. Figures to the right indicate marks allotted.
All symbols have their usual meaning.

Q.1	**(A)**	(i)	Prove that $\vec{\nabla}\left(\frac{\vec{r}}{r^n}\right) = \frac{3-n}{r^n}$ for ($n > 0$) and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.	[07]
		(ii)	Derive expression of velocity and acceleration in spherical polar coordinate system.	[07]
			OR	
	(A)	(i)	Show that Laplacian operator in spherical coordinate system is given as $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$.	[07]
		(ii)	Describe cartesian tensor and Einstein's summation convection.	[07]
	(B)		Attempt any four questions :	[04]
		(i)	In which coordinate system unit vectors always point in the same direction and do not change direction from point to point?	
		(ii)	Write differential length ($d\vec{r}$) for cartesian and cylindrical system.	
		(iii)	Obtain $\frac{\partial\hat{r}}{\partial\theta}$ in spherical polar coordinate system.	
		(iv)	Differentiate between covariant and contravariant form of tensor.	
		(v)	Differential element of length transforms ______ and gradient of scalar transforms ______.	
		(vi)	Define tensor of rank 0. Do physical observables depend on the choice of coordinate frames or changes with the choice of coordinate frames?	
Q.2	**(A)**	(i)	Solve $x^2y'' + 5xy' + (-x+3)y = 0$ by Frobenius method.	[07]
		(ii)	Evaluate $\int_0^{2\pi}\frac{\cos 3\theta}{-4\cos\theta+5}d\theta$ where $\lvert z\rvert = 1$.	[04]
		(iii)	Find the residue of $f(z) = \frac{e^z}{z^2+a^2}$.	[03]
			OR	
	(A)	(i)	Solve $x^2y'' + xy' + (-1+x^2)y = 0$, given $y_1(x) = J_1(x)$.	[07]
		(ii)	Find the residue of $f(z) = \frac{1}{(z^2+1)^3}$ at $z = i$.	[04]
		(iii)	Evaluate $\int_c \frac{e^z(z^2+1)}{(z-1)^2}$ dz, where C is a circle $\lvert z\rvert = 2$.	[03]
	(B)		Attempt any four questions :	[04]
		(i)	Is $z^2\bar{z}$ an analytic function?	
		(ii)	Define a regular point.	
		(iii)	Give difference between simple pole and multiple pole?	
		(iv)	Define: Mapping.	
		(v)	The transformation $w = z^5 - 5z$ fails to be conformal at ______.	
		(vi)	State the Cauchy's theorem.	
Q.3	**(A)**	(ii)	Express polynomials $f(x) = 4x^3 + 6x^2 + 7x + 2$ and $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials.	[07]
		(ii)	Obtain generating function for Bessel's polynomial.	[07]

			OR	
	(A)	(i)	Derive the recurrence formulae for Legendre's polynomials are $(n+1)P_{n+1}(x) = (2n+1)\, x\, P_n(x) - nP_{n-1}(x)$ and $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$.	[07]
		(ii)	Solve $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$ using power series method.	[07]
	(B)		Attempt any three questions :	[03]
		(i)	Bessel's function $J_n(x)$ is _________ function when n is even and is ________ function when n is odd.	
		(ii)	$J_1(x=0)$ is __________(regular/singular – choose correct option)	
		(iii)	For Hermites polynomials $\int_{-\infty}^{\infty} e^{-x^2} H_m(x)H_n(x)dx =$________, when $m \neq n$.	
		(iv)	Write Rodrigue's formula for Legendre polynomials and from that prove $\int_{-1}^{1} P_0(x)dx = 2$.	
		(v)	For Laguerre function $L_1(x) =$ _________.	
Q.4	**(A)**	(i)	Prove convolution theorem. Find $L^{-1}\left\{\frac{1}{(s+1)^2(s-2)}\right\}$.	[07]
		(ii)	Find the Laplace transform of following functions: (a) $f(t) = \frac{t}{k}$; $0 < t < k$ $= 1$; $t > k$, (b) $f(t) = t - 1$; $1 < t < 2$ $= 3 - t$; $2 < t < 3$. Here, k is constant.	[07]
			OR	
	(A)	(i)	Find Fourier transform of $f(t)$ if $f(t) = 1$; $\lvert x\rvert < a$ $= 0$; $\lvert x\rvert > a$ and hence evaluate $\int_{-\infty}^{\infty} \frac{\sin sa \cos st}{s} ds$ and $\int_{0}^{\infty} \frac{\sin s}{s} ds$.	[06]
		(ii)	Show that $F_s\{t\, f(t)\} = -\frac{d}{ds} F_c\{f(t)\}$ and $F_c\{t\, f(t)\} = \frac{d}{ds} F_s\{f(t)\}$.	[03]
		(iii)	Find Fourier sine and cosine transform of te^{-at}.	[05]
	(B)		Attempt any three questions :	[03]
		(i)	For the unit impulse function $H(t-a)$, the value of $L\{H(t-a)\} =$_______.	
		(ii)	Laplace transform of function $(t) = t$, where $t \geq 0$ is_______. (a) s (b) $1/s$ (c) $1/s^2$ (d) $1/s^3$	
		(iii)	Find the Laplace transform of function $\cos t \log t\, \delta(t-\pi)$.	
		(iv)	If $L\{f(t)\} = f(s)$ then $L\{(f(t)''\} =$__________.	
		(v)	Fourier transform is used for solving _________. (a) boundary value problem (b) partial differential equations (c) integrals (d) (a), (b) & (c)	