M.Sc. Physics Examination

Semester – I

Paper No – C103 Mathematical Methods in Physics

Paper Code – 4514

Time: 2 Hours 30 min NOV – 2015 Maximum Marks 70

Notes: (1) - All questions are compulsory. (2) Number in square bracket indicate marks

Q.1 (a) Calculate (i) ∇r (ii) ∇·**r** and (iii) ∇x**r**. [6]

(b) A potential is given in cylindrical coordinates as $k/\sqrt{\rho^2 + z^2}$. Find the force field it represents and express the field in spherical polar coordinates. [8]

OR

(a) Express the Cartesian component of dell (**∇**) operator in cylindrical coordinates. [8]

(b) Find the gradient of

(i) $x^3 + y^3 + z^3$ (ii) $e^{-\rho^2}$ (iii) $r^3 \cos\theta \ \cos\phi$ [6]

Q. 2 (a) State and prove Cauchy's integral Theorom. [7]

(b) Evaluate the following integral using residue theorem [7]

$$\int_0^{2\pi} \frac{d\theta}{10 - 8\ \cos\theta}$$

OR

(a) Find the residues of the following functions at given points [9]

$$(i)\frac{\sin z}{(1 - z^4)}\ at\ z = i \quad (ii)\frac{z}{(2z-1)(5-z)}\ at\ z = \ -\frac{1}{2}, 5$$

(b) Prove that $u = \ e^{-x}(x\ \sin y - y\ \cos y)$is harmonic. [5]

Q. 3 (a) Prove following recursion relation for Legendre' Polynomial [7]

$$(n+1)P_{n+1}(x) - x(2n+1)P_n(x) + nP_{n-1}(x) = 0$$

(b)Obtain the generating function for Bessel's polynomials. [7]

OR

(a) Show that [5]

$$\frac{2}{3}P_2(x) + \frac{1}{3}P_0(x) = x^2$$

(b) Solve the differential equation by changing the independent variable method [5]

$$x\frac{d^2y}{dx^2} + (4x^2 - 1)\frac{dy}{dx} + 4x^3y = 2x^3$$

(c) Using Rodrigue's formula, prove that [4]

$$\int_{-1}^{+1} P_n(x)\, dx = 0, (n \neq 0)$$

Q. 4 (a) Solve the given simultaneous differential equations for $x(t)$ and $y(t)$ with the help of Laplace transform methods

$$\frac{dx}{dt} + x + 4y = 10 \quad \& \quad x - \frac{dy}{dt} - y = 0$$

Given that $x(0) = 4, y(0) = 3$ [10]

(b) Write formula for generating function of Hermite and Laguerre functions. [4]

OR

(a) Using the method of separation of variables obtain the solution of a wave equation applicable to the spherical membrane. [8]

(b) Show that the value of the integral [6]

$$\int_{-1}^{+1} x^2 P_3(x)\, dx = 0$$

Q.5 (a) Represent a function $f(x)$ as [14]

$$f(x) = \begin{cases} 1 & for \;\; 0 < x < \frac{1}{2} \\ 0 & for \;\; \frac{1}{2} < x < 1 \end{cases}$$

by (a) Fourier Sine series

(b) Fourier Cosine series

(c) Fourier Series

(d) Confirm the results of Fourier series with the help of Complex form of Fourier Series and sketch the graph of function for each condition.

OR

(a) The displacement of a damped harmonic oscillator as function of time is given by

$$f(t) = \begin{cases} 0 & for \;\; t < 0 \\ e^{-t/\tau} \sin \omega_0 t & for \;\; t \geq 0 \end{cases}$$

Show that the Fourier transform of this function is given as [7]

$$F(\omega) = \frac{1}{2}\left[\frac{1}{\omega + \omega_0 - \frac{i}{\tau}} - \frac{1}{\omega - \omega_0 - \frac{i}{\tau}}\right]$$

(b) If Fourier transform (F.T.) of $f(x)$ is $F(\alpha)$ then by using the Fourier transform of derivatives show that [7]

$$\text{F. T.}\{f^n(x)\} = (i\alpha)^n F(\alpha)$$