19 FEB 2021

M.Sc. Statistics (Sem I) Examination -2021 **Code: 22937**

Paper – 103: Inference-I (Theory of Estimation)

Time: $1\frac{1}{2}$ hour **Total marks: 42**

Attempt any three questions.

1. (a) Explain unbiasedness, consistency, efficiency and sufficiency. State and prove sufficient conditions for consistency. **14**

OR

1. (a) **(i)** State and prove Rao-blackwell theorem. 7

1 (a) **(ii)** Let $x_1, x_2, \ldots . x_n$ be a random sample from $P(x) = p^x(1-p)^{1-x}$, $x = 0,1,2 \ldots . n, 0 < p < 1$, than show that $T = \sum x_i$ is sufficient statistic for p. 7

2. (a) State and prove Cramer-Rao inequality. Find Cramer-Rao lower bound for the variance of any unbiased estimator of $\varphi(\theta) = \theta^2$ based on a random sample from a poisson distribution with mean θ. **14**

OR

2. (a) **(i)** State and prove Neyman Fisher Factorization theorem. 7

2 (a) **(ii)** State and prove Bhattacharya inequality. 7

3. (a) Explain the method of moments, also state its merits and demerits. Let $x_1, x_2, \ldots . x_n$ be a random sample from $f(x, \theta) = \theta e^{-\theta x}$, $0 < x < \infty, \theta > 0$, estmate θ by method of moments. **14**

OR

3. (a) **(i)** Explain the method of modified minimum chi-square for estimation. 7

3 (a) **(ii)** State the properties of MLE. 7

4. (a) Explain Prior distribution and posterior distribution. Define Baye's estimator. A random sample from Binomial(n, θ) and prior distribution of θ be Beta(α, β), $0 < \theta < 1$, then obtain Baye's estimator under SEL. **14**

OR

4. (a) **(i)** A random sample from $f(x, \theta) = \frac{e^{-\theta}\theta^x}{x!}$ and prior distribution of θ be gamma(α, β), $g(\theta) = \frac{\beta^\alpha}{\Gamma\alpha} x^{\alpha-1} e^{-\beta x}$, $0 < \theta < \infty$, then obtain Baye's estimator under SEL. 7

4 (a) **(ii)** Explain: 7

(i) Subjective probability

(ii) Squared error loss function and weighted squared error loss function

(iii) Risk function and Baye's risk