M.Sc.(Sem. –I) EXAMINATION

STATISTICS :PARER 101

Sub. code - 22935 linear algebra

TIME :1 ½ Hours. 17 FEB 2021 TOTAL MARKS:42

Instructions:

1. There are FOUR Questions in this paper. Attempt ANY THREE
2. Each question carries 14 marks.
3. Use of scientific calculator is allowed.

1. (a) Define characteristic root and characteristic vector of a matrix , determine characteristic roots, algebraic and geometric multiplicity of characteristic root of given matrix **14**

$$\begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & 2 \end{bmatrix}$$

OR

1. (a) Prove that A set of orthogonal vectors is always L.I. **7**

(b) Using Gram Smidth Orthogonolization process, construct orthogonal basis form from {(1,2,3),(1,3,4),(3,4,5)}. **7**

2. (a) Show that **14**

(i)$\bar{A}$ exist iff H=$\bar{A}A$ is idempotent

(ii) r(H)=r(A)=trace(H).

(iii) r(AB)≤{r(A),r(B)}

OR

2 (a) If A:m×m, B: m×n, C: n×m and D: m×n are matrices and P:(m+n)×(m+n) is a non-singular matrix such that P=$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ then show that **7**

$P^{-1}=\begin{bmatrix} R & -RBD^{-1} \\ -D^{-1}CR & D^{-1}+D^{-1}CRBD^{-1} \end{bmatrix}$ if D is non-singular and

$R=(A-BD^{-1}C)^{-1}, P=|D||A-BD^{-1}C|$

(b) Show that matrix A: m×n is idempotent matrix if and only if Rank (A) + Rank (I-A) = n. **7**

3. (a) Let A: m×n be any matrix and $\bar{A}$:n×m be a g-inverse of the matrix A. Further, let H=$\bar{A}A$ and H :n×n the prove that **14**

   i) A general solution of the homogenous system of linear equation Ax=0 is X=(I-H)z. where Z is any arbitrary constant, OR

3 (a) The necessary and sufficient condition for non-homogenous equation Ax=b to be consistent if r (A,b)=r(A). **7**

(b) Let Ax=o be the given system of linear homogenous equation if rank (A)= r then there exists (n-r) linearly independent solution to the system of equation. **7**

4. (a) Explain criterion for positive definiteness of Quadratic form in terms of leading principal minor. Also give classification of quadratic form in terms of leading principal minor. **14**

Is the following quadratic form positive definite?

$Q = 6X_1^2 + 35X_2^2 + 11X_3^2 - 4X_1X_3.$

OR

4 (a) Define Real Quadratic Form, diagonal form, Jordan block and Jordan matrix. Give classification of Quadratic form. Find the matrix A of each of the following bilinear form $f(X,Y)=X^TAY$ **7**

$3x_1y_1+x_1y_2-2x_2y_1+3x_2y_2-3x_1y_3$

(b) Reduce the following quadratic form to canonical form by congruent transformation. Also find Rank, Index, Signature and class value. **7**

$Q = 6X_1^2 + 3X_2^2 + 3X_3^2 - 2X_2X_3 + 4X_1X_3 - 4X_1X_2$