M.Sc.(Sem. –I) EXAMINATION
STATISTICS :PARER I - 2752
linear algebra -

TIME : _____   TOTAL MARKS:70

Note: (i) All Questions are Compulsory

(ii) All Full Questions carry equal marks.

1. (a) Explain Gram Smidth Orthogonolization process.   **14**

Consider $S=\{X_1=\binom{3}{1}, X_2 = \binom{2}{2}\}$ set of vectors in R^2. Obtain an orthogonal set of vectors.

OR

1. (a) Determine characteristic roots, algebraic and geometric multiplicity of   **7**
characteristic root of given matrix

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

(b) Show that $\{(1,0,-1),(1,2,1),(0,-3,2)\}$ is basis of $V_3(R)$.   **7**

1. B **Attempt any FOUR**   **4**
   (i) Give the condition for AX=b is solvable.
   (ii) Given that A is any Matrix of order n×n. Sum of its characteristics roots is 8. What will be trace of A?
   (iii) If rank of Orthogonal matrix A is 4 then what will be the characteristic root of given matrix?
   (iv) If Characteristic roots of square Matrix A is 1,4,6 then what will be trace of A?
   (v) A is orthogonal matrix then what will be characteristic roots of A.
   (vi) If Characteristic roots of square Matrix A is 0,1,2,3 then A is_____.

2 (a) If A:m×m, B: m×n, C: n×m and D: n×n are matrices and   **7**
P:(m+n)×(m+n) is a non-singular matrix such that P=$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ then show that

$P^{-1}=\begin{bmatrix} A^{-1} + A^{-1}BQCA^{-1} & -A^{-1}BQ \\ -QCA^{-1} & Q \end{bmatrix}$ if A is non-singular and Q=(D − $CA^{-1}B)^{-1}$.

(b) Prove that for any 2 matrices A and B   **7**
Rank (AB) = Rank (BA) = Rank(A) where B is non-singular.

OR

2. (a) Show that matrix A: m×n is idempotent matrix if and only if **14**
Rank (A) + Rank (I-A) = n.
Also Prove that Rank (A) + Rank (I-A) –n = Rank [(I-A) A].

2. **B** **Attempt any FOUR.** **4**

(i) Let B be a non-singular matrix and Rank of A is 3. What will be rank of AB?

(ii) A is idempotent matrix then give the relation of rank and trace of matrix A.

(iii) Fill up the blank: Rank[(A(In-A)]=Rank(A)+_____-n.

(iv) Matrix A is idempotent matrix and sum of characteristic roots of A is 4 then what is the rank of Matrix A?

(v) State the relation between Rank(A,B) ____ Rank (A) + Rank(B)

(vi) If matrix A has 3 linearly independent columns, what will be rank of matrix A?

3. (a) Prove that A^{-1} exists if and only if $A\bar{A}A=A$. Also let $\bar{A}$ be any g-inverse **14**
of A and $H=\bar{A}A$, then prove that general solution of a consistent non-homogenous equation Ax=y is $x=\bar{A}y + (i-H)z$, where z is any arbitrary vector.

OR

(a) Define: The Moore-Penrose g-inverse of a matrix. Prove that Moore-Penrose is unique. **7**

(b) Let $\bar{A}$ be any g-inverse of A and $H=\bar{A}A$, then prove that general **7**
solution of a consistent non-homogenous equation.

3. **B** **Attempt any THREE** **3**

(i) G is Moore-Penrose inverse of A then give the relation between G and A

(ii) G is regular inverse of A then give the relation between G and A.

(iii) If ABA=A, Mention the relation between A and B.

(iv) x=Gb is one solution of Ax=b then what will be the condition of A?

(v) What is the general solution of homogeneous linear equation and non-homogenous liner equation?

4. (a) Define Real Quadratic Form. Explain Types of Quadratic form. **14**

OR

(a) Explain Sylvester's Criterion for positive definite form. Is the matrix **7**
$A=\begin{bmatrix} 1 & 3 & -2 \\ 3 & 2 & 2 \\ -2 & 2 & 4 \end{bmatrix}$ positive definite? Justify your Answer.

(b) Reduce the following quadratic form to canonical form by congruent transformation. Also find Rank, Index, Signature and class value. **7**

$$Q = X_1^2 + 2X_2^2 + 7X_3^2 - 2X_1X_2 + 4X_1X_3 - 6X_2X_3$$

4. B **Attempt any Three**

4. (i) Define: **3**

Signature and Index of Real Quadratic Form

(ii) Let Q be Quadratic Form and rank of Q = signature of Q=index of Q, then Q will be____.

(iii) Mention the type of quadratic form when $5Y_1^2 + Y_2^2$ are given.

(iv) A four variable quadratic form has rank 3 and index 2 then quadratic form will be of which type.

(v) Given that $-x^2 - y^2 - z^2$, mention the type of given quadratic form.