

Code :- 2752

M.Sc.(Sem. -I) EXAMINATION - Nov- 2015

STATISTICS :PARER 01

Linear Algebra

TIME : Two Hours.

TOTAL MARKS:70

Note: (i) All Questions are Compulsory

(ii) All Full Questions carry equal marks.

1. (a) For any two matrices $A_{p \times m}$ and $B_{p \times n}$ prove that 8
 $\text{rank}(A, B) \leq \text{Min}[\rho(A), \rho(B)]$
(b) Explain (i) linearly independent (ii) linearly dependent 6
(iii) basis (iv) orthogonal vector

OR

1. (a) Show that square matrix A is non-singular iff all its 8
columns are linearly independent
(b) For any two matrices $A_{p \times m}$ and $B_{p \times n}$ prove that 6
 $\text{Max}[\rho(A), \rho(B)] \leq \text{rank}(A, B) \leq \text{rank} A + \text{rank} B$
2. (a) If $P = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is a partitioned nonsingular matrix 8
where A is non
Singular then derive P^{-1}
(b) 6
Show that the system of linear non-homogeneous equation $A\underline{x} = \underline{b}$
is consistent, if $\rho(A, b) = \rho(A)$.

OR

2. (a) Show that for matrix $A_{n \times n}$ 8
 $\text{Rank}(A) + \text{Rank}(I - A) - n = \text{Rank}((I - A)A)$
(b) For any two matrices A and B prove that 6
 $\rho(AB) = \rho(BA) = \rho(A)$, B is non-singular.

3. (a) Show that matrix $A_{m \times n}$ is idempotent matrix iff $\text{Rank}(A) + \text{Rank}(I-A) = n$ 6
 (b) Show that \bar{A} exists iff $A \bar{A} A = A$ 8

OR

- 3 (a) Explain types of g-inverse 6
 (b) Reduce the symmetric matrix 8

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{to a diagonal matrix D.}$$

4. (a) 8
 Let \bar{A} be any g-inverse of A and $H = \bar{A} A$, then prove that general solution of a consistent non-homogeneous equation $Ax=y$ is $x = \bar{A} y + (I - H)z$, z is any arbitrary vector.
 (b) Show that \bar{A} exists iff $H = \bar{A} A$ is idempotent 6

OR

4. (a) Write a note on (i) Types of Quadratic form 8
 (b) Show that $\{(1,1,0), (1,0,1), (0,1,1)\}$ is a basis of $V_3(\mathbb{R})$.
 5 (a) Explain Gram Smith orthogonalization process 6
 (b) Reduce the quadratic form 8

$Q = x_1^2 + 2x_2^2 - x_3^2 - 12x_1x_2 - 4x_1x_3$ to its diagonal forms and determine its types.

OR

- 5 (a) Define Moore-Penrose g-inverse of a matrix. Prove that it is unique. 8
 (b) Define following terms 6
 (i) equivalence of Quadratic forms
 (ii) Quadratic forms (iii) Rank of Quadratic form
 (iv) Normal form of Quadratic form
 (v) Signature of Quadratic form (vi) Diagonal form of a Quadratic form.