26 OCT 2020

: નોંધ : M.Sc (Maths) Sem. II Complex Analysis Sub. Code. 3115

૧. દરેક પ્રશ્નનો [a] અથવા [a(i)] અને [a(ii)] જ લખવાના રહેશે.
૨. પ્રશ્ન : ૧[a] અથવા ૧[a(i)] અને ૧[a(ii)] તથા ૨[a] અથવા ૨[a(i)] અને ૨[a(ii)] ના 14 માર્કસ ના બદલે ૧૮ માર્કસ રહેશે.
૩. પ્રશ્ન : ૩[a] અથવા ૩[a(i)] અને ૩[a(ii)] તથા ૪[a] અથવા ૪[a(i)] અને ૪[a(ii)] ના 14 માર્કસ ના બદલે ૧૭ માર્કસ રહેશે.
૪. દરેક પ્રશ્નનો પ્રશ્ન નં ૧(b), પ્રશ્ન નં ૨(b), પ્રશ્ન નં ૩(b) તથા પ્રશ્ન નં ૪(b) (ટુંકા પ્રશ્નો) વિદ્યાર્થીએ લખવાના નથી.

Q.1 (A) If n is a rational number then prove that one of the value of $(cos\theta + isin\theta)^n$ is $\cos n\theta + i \sin n\theta$. [14]

OR

Q.1 (A) (i) If $x_n = cis\frac{\pi}{3^n}$. Prove that $i.(x_1.x_2.x_3 \dots x_\infty) = -1$. [7]

(ii) If $\text{Cos}\,\alpha + \text{Cos}\,\beta + \text{Cos}\,\gamma = 0 = \text{Sin}\,\alpha + \text{Sin}\,\beta + \text{Sin}\,\gamma$. Then prove that $\text{Cos}\,2\alpha + \text{Cos}\,2\beta + \text{Cos}\,2\gamma = 0$ [7]

Q.1 (B) **Attempt any four out of six.** [4]

(1) Is $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$?
(2) Write polar form of $z = 1 + i$.
(3) Write triangle inequality for $|z_1 + z_2|$.
(4) For $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$, write the $Arg(z+1)$.
(5) Write solutions of $z^3 + 1 = 0$.
(6) Is n^{th} roots of unity are in geometric progression?

Q.2 (A) Find polar form of Cauchy – Riemann equations. Using it show that $W = \log Z$ is analytic in the complex plane except at the origin. [14]

OR

Q.2 (A) (i) Find the analytic function of which the real part is $e^{-x}[(x^2 - y^2)\cos y + 2xy \sin y]$. [7]

(ii) Show that $u = x^2 - y^2 - y$ is harmonic and determine the corresponding analytic function $f(z) = u + iv$. [7]

Q.2 (B) **Attempt any four out of six.** [4]

(1) Define Harmonic function.
(2) Define Analytic function.
(3) State C – R equation in Cartesian coordinate.
(4) Define bilinear transformation.
(5) Define inversion transformation.
(6) Is i^i is real?

Q.3 (A) **(i)** Find the bilinear transformation which maps the point $z = 1, i, -1$ onto the points $w = i, 0, -i$. [14]

(ii) Determine the bilinear transformation that maps $z_1 = 0,\ z_2 = 1, z_3 = \infty$ onto $w_1 = -1,\ w_2 = -i, w_3 = 1$ respectively.

OR

Q.3 (A) (i) Show that the transformation $w = \frac{1}{z}$ maps the circle $|z - a| = a$, [7]

$(a \neq 0)$ in z plane to a straight line in w – plane.

(ii) Find the image of the circle $(x-3)^2 + y^2 = 4$ under the transformation $W = \frac{1}{z}$. [7]

Q.3 (B) **Attempt any three out of five.** [3]

(1) Define the Transformation Translation.

(2) Define invariant point.

(3) Write invariant point of transformation $W = \frac{1}{z}$.

(4) Write the fixed point of transformation $W = \frac{iz}{2}$.

(5) Evaluate $\oint_c \frac{1}{z^3}\, dz,\ C: |z| = 1$.

Q.4 (A) State and prove Cauchy's integral formula. [14]

OR

Q.4 (A) (i) Evaluate $\oint_c \frac{z^2+1}{z^2-1}\, dz$,where C is the circle of unit radius with centre at $z = 1$. [7]

(ii) Evaluate $\oint_c \frac{4-3z}{z(z-1)(z-2)}\, dz$,where C is the circle $|z| = \frac{3}{2}$. [7]

Q.4 (B) **Attempt any three out of five.** [3]

(1) State Liouville's theorem.

(2) State Cauchy's inequality.

(3) State fundamental theorem of algebra.

(4) For $f(z) = \frac{2z+1}{z^2-z-2}$, write the value of $\underset{at\ z=-1}{Res} f(z)$.

(5) Write the simple poles of $f(z) = \frac{1}{z^2+1}$.

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