Code: 3115

APRIL 2015

M.Sc. Mathematics Paper No: 8
Semester- II
Complex Analysis

Marks – 70

Q-1(A) Find distinct values of $(1+i)^{\frac{1}{3}}$ (7)

(B) State DeMoiver's Theorem. Find fifth roots of -1 (7)

OR

Q-1(A) If $i^{i^{i.....\infty}} = A + iB$ Prove that $A^2 + B^2 = e^{-\pi(4n+1)B}$ (7)

(B) If $x_r = \cos\frac{\pi}{2^r} + i\ \sin\frac{\pi}{2^r}$ Prove that $x_1 . x_2 . x_3 ....\infty = -1$ (7)

Q-2(A) Show that the function f(Z) = xy + iy is continuous everywhere but is not harmonic. (7)

(B) Prove that real and imaginary part of an analytic function satisfy Laplace's equation. (7)

OR

Q-2(A) Show that C-R equations for the function $\omega = \log Z$ are satisfied everywhere except at origin. (7)

(B) Show that the function $u(x,y) = e^x \cos y$ is harmonic. Determine its harmonic conjugate. (7)

Q-3(A) An electroststic field in the XY plane is given by the potential function $\phi = 3x^2y - y^3$ find the stream function. (7)

(B) Prove that every convergent sequence in C is a Cauchy sequence. (7)

OR

Q-3(A) Discuss the following transformations (7)

(i) $w = z^2$ (ii) Bilinear transformations (iii) $w = e^Z$ (iv) $w = \frac{1}{2}$

(B) Find the image of infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ (7)

Q-4(A) Find the image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$ (7)

(B) Find the image of the region bounded by $1 \le r \le 2$ and $\frac{\pi}{6} \le \theta \le \frac{\pi}{3}$ in the Z plane under the transformation $w = z^2$. (7)

OR

Q-4(A) State and prove Morera's Theorom (7)

(B) State and prove Liouville's Theorem (7)

Q-5(A) State and prove Cauchy's Theorem (7)

(B) Evaluate $\oint_C \frac{dz}{z-3}$ where $c: |z-2| = 5$ (7)

OR

Q-5(A) Evaluate $\oint_C \frac{z+4}{z^2+2z+5} dz$ where $c: |z+1| = 1$ (7)

(B) If $f(z) = \frac{z}{(z-1)(z+4)}$ then evaluate Res (f(z),1) (7)