M.Sc. Semester – 2
April 2015 Examination
Paper – 6: Linear Algebra

Time: 2 ½ Hours Code – 4758 **Total Marks: 70**

Q.1 (a) Let V be a vector space over R then prove that [7]

(i) $0 \cdot x = \theta$ (ii) $(-1)\cdot x = -x \quad \forall x \in V$ (iii) $\alpha \cdot \theta = \theta \quad \forall \alpha \in R$

(b) Check whether the following subsets of R^2 are vector subspace or not? [7]

(i) $\{ (x, y) \in R^2 \mid x_1 x_2 = 0 \}$ (ii) $\{ (x, y) \in R^2 \mid 3x_1 + 2x_2 = 0 \}$

OR

Q.1 (a) Find all solutions to the following system of equations by row-reducing the coefficient matrix: [7]

$$\frac{1}{3}x_1 + 2x_2 - 6x_3 = 0$$

$$-4x_1 \qquad + 5x_3 = 0$$

$$-3x_1 + 6x_2 - 13x_3 = 0$$

$$\frac{-7}{3}x_1 + 2x_2 - \frac{8}{3}x_3 = 0$$

(b) Prove that any basis of a vector space is linearly independent set. [7]

Q.2 (a) State and prove Laplace expansion: [7]

(b) Find matrix associated with the following linear transformations with respect to the standard basis: [7]

(i) T: $R^2 \to R^3$ defined as T(x, y) = (x, x + y, y), $\forall (x, y) \in R^2$

(ii) T: $R^3 \to R^3$ defined as T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z), $\forall (x, y, z) \in R^3$

OR

Q.2 (a) Check whether the following maps are linear or not? [7]

(i) T: $R^2 \to R^2$ defined as T(x, y) = (x – 2, y), $\forall (x, y) \in R^2$.

(ii) T: $R^3 \to R^3$ defined as T(x, y, z) = (x + y + z, 2x + y, x – y + z), $\forall (x, y, z) \in R^3$.

(b) Using definition, find det(A) for the matrix $A=\begin{pmatrix} 1 & 5 & 0 & 0 \\ 2 & 0 & 8 & 0 \\ 3 & 6 & 9 & 0 \\ 4 & 7 & 10 & 1 \end{pmatrix}$ [7]

Q.3 (a) Explain the following terms: [7]

(i) Monic polynomial (ii) Triangulation (iii) Direct sum of matrices.

(b) Find eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$. [7]

OR

Q.3 (a) State and prove Cayley – Hamilton theorem. [7]

(b) Let V be a finite dimensional inner product space. If T and U are linear operators on V and $\alpha \in R$ then prove that [7]

(i) $(\alpha T)^* = \alpha T^*$ (ii) $(TU)^* = U^* T^*$ (iii) $(T^*)^* = T$.

Q.4 (a) Check whether the following products are inner product on V or not? [7]

(i) $V = R^2$, $\langle x, y\rangle = y_1 (x_1 + 2x_2) + y_2 (2x_1 + 5x_2)$, where $x = (x_1, x_2)$, $y = (y_1, y_2) \in R^2$.

(ii) $V = R$, $\langle x, y\rangle = |x - y|$, $\forall x, y \in V$

(b) Obtain orthogonal set from the set { (–1, 0, 1), (1, –1, 0), (0, 0, 1) }. [7]

OR

Q.4 Prove that every inner product space has an orthonormal basis. [14]

Q.5 (a) Define: (i) Adjoint of a linear operator (ii) Self – adjoint operator (iii) Unitary operator (iv) Normal operator [7]

(b) Let V be a finite dimensional inner product space and T be a normal operator on V. Suppose $v \in V$. Then prove that v is a characteristic vector for T with characteristic value λ if and only if v is a characteristic vector for T^* with characteristic value $\bar{\lambda}$. [7]

OR

Q.5 (a) Let U be a linear operator on an inner product space V. Prove that U is unitary if and only if U^* exists and $UU^* = U^*U = I$. [7]

(b) State and prove spectral theorem. [7]
