17 AUG 2020

M.Sc. (Physics) Sem. II

Sub Code: 4657

: નોંધ :

૧. દરેક પ્રશ્નનો [a] અથવા [a(i)] અને [a(ii)] જ લખવાના રહેશે.
૨. પ્રશ્ન : ૧[a] અથવા ૧[a(i)] અને ૧[a(ii)] તથા ૨[a] અથવા ૨[a(i)] અને ૨[a(ii)] ના 14 માર્કસ ના બદલે ૧૮ માર્કસ રહેશે.
૩. પ્રશ્ન : ૩[a] અથવા ૩[a(i)] અજે ૩[a(ii)] તથા ૪[a] અથવા ૪[a(i)] અને ૪[a(ii)] ના 14 માર્કસ ના બદલે ૧૭ માર્કસ રહેશે.
૪. દરેક પ્રશ્નનો પ્રશ્ન નં ૧(b), પ્રશ્ન નં ૨(b), પ્રશ્ન નં ૩(b) તથા પ્રશ્ન નં ૪(b) (ટુંકા પ્રશ્નો) વિદ્યાર્થીએ લખવાના નથી.

Note: Answer all questions. Figures to the right indicate marks allotted.
All symbols have their usual meaning.

1	a)	i) Define expectation value of any physical observable. Give physical interpretation of eigenvalues. Define A-representation for dynamical quantity. ii) Write and prove postulate-**4** of quantum mechanics. Write physical meaning conveyed by it.	07 07
		OR	
	a)	i) Define *raising* $(a^{\dagger})$ and *lowering* (a) operators for SHO. Obtain Hamiltonian operator for SHO in terms of these operators. Obtain energy eigen spectrum and eigenvectors. ii) Give detailed note on matrix representation of any dynamical quantity/operator, $\hat{F}$. Prove that in its own representation, matrix representing $\hat{F}$ is *diagonal*.	07 07
	b)	Attempt any four questions. i) Prove that $[x^2, p^2] = -2\hbar^2 + 4i\hbar px$. ii) Prove that $\langle AA^{\dagger}\rangle$ is always real. iii) Prove that $\vec{L} \times \vec{L} = i\hbar\vec{L}$. iv) Define projection operator. v) For two operator A and B, prove that $(A + B) = (B + A)$.	04
2	a)	i) In Rayleigh–Schrödinger perturbation theory for non-degenerate case, discuss the problem of solving second-order energy correction term. Explain the method of Dalgarno–Lewis taking an example of H-atom in a uniform electric field. ii) When H-atom is placed in a uniform electric field, discuss in details the first order correction the first excited state. Derive energy eigenstate for any one eigenvalue. Given: $\vert u_{200}\rangle = N\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}$, $\vert u_{21-1}\rangle = N\left(\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}} sin\theta\, e^{-i\varphi}$, $\vert u_{210}\rangle = N\left(\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}} cos\theta$, $\vert u_{211}\rangle = -N\left(\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}} sin\theta\, e^{+i\varphi}$; where normalization constant, $N = \left(\frac{1}{32\pi a_0^3}\right)^{\frac{1}{2}}$ and Bohr radius $a_0 = \frac{\hbar^2}{me^2}$.	07 07
		OR	
	a)	i) Obtain ground state energy of He-atom using variation technique. How variation technique is extended to excited states? ii) Consider the trial wave function $\varphi(r) = Ae^{-\alpha r}$ as the ground state of H-atom taking α as variation parameter and A is normalization constant. Derive ground state energy and optimized wave function for H-atom.	07 07
	b)	Attempt any four questions. i) For one dimensional *anharmonic* oscillator PE is given by $V(x) = \frac{1}{2}kx^2 + ax^3$. What will be the first order correction to the energy eigenvalue? Why? Given: Eigenket for SHO is $\vert u_n\rangle = Ne^{-\frac{1}{2}\rho^2}H_n(\rho)$; $n = 0, 1, 2, \ldots$ $H_n(\rho)$ is the Hermite polynomial and N is the normalization constant. ii) Energy eigenvalues of Li-atom are computed assuming its valency as a variation parameter Z'. With this presumption, write its Hamiltonian. iii) Give difference between perturbation technique and variation principle.	04

		iv) The ground state of a one-dimensional anharmonic oscillator, with its Hamiltonian given by $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + bx^4$, is given by Gaussian trial wave function $\varphi(x) = Ae^{-\alpha x^2}$. Find normalization constant A. Given: $\int_0^\infty e^{-px^2}dx = (\frac{\pi}{p})^{\frac{1}{2}}$ and $\int_0^\infty x^4 e^{-px^2}dx = \frac{3\sqrt{\pi}}{8}p^{-\frac{5}{2}}$. v) For variation method, a criterion for energy minimization is given by ______. Select the correct option from below. (A) $[\langle\psi\|H^2\|\psi\rangle - W^2]^{\frac{1}{2}} \geq (W - E_0)$ (B) $[\langle\psi\|H^2\|\psi\rangle + W^2]^{\frac{1}{2}} \geq (W - E_0)$ (C) $[\langle\psi\|H^2\|\psi\rangle - W^2]^{\frac{1}{2}} \leq (W - E_0)$ (D) $[\langle\psi\|H^2\|\psi\rangle + W^2]^{\frac{1}{2}} \leq (W - E_0)$	
3	**a)**	i) For time-dependent perturbation $H'(t)$, derive an expression for transition amplitude $a_f^{(1)}(t)$, and give its physical interpretation. ii) For t-dependent perturbation, first-order transition amplitude is given by an expression $a_f^{(1)}(t) = \frac{1}{i\hbar}\int_{t'=0}^{t} H'_{fi}(t')\, e^{i\omega_{fi}t'}dt'$. For constant perturbation discuss *transition probability* and for the case of closely spaced energy levels, derive Fermi-Golden rule.	07 07
		OR	
	a)	i) Derive asymptotic solution for one-dimensional Schrödinger equation using WKB method. ii) Write note on *propagator*. For a case of harmonic perturbation, derive the expression for transition amplitude. Here, $\hat{H}'(\vec{r};t) = 2\hat{H}'(\vec{r})\cos(\omega t)$.	07 07
	b)	Attempt any three questions. i) Give difference between retarded and advanced form of *propagator*. ii) Choose the correct relation from below. At the *turning point* ______. (A) E = P.E. and K.E. = 0 (B) E = K.E. and P.E. = 0 (C) E > K.E. and P.E. = 0 (D) E > P.E. and K.E. = 0 iii) Give Bohr-Sommerfeld quantization condition. iv) Why *propagator* is known as *propagator*?	03
4	**a)**	i) Describe wave mechanical picture of scattering phenomena, and deduce relation between scattering amplitude $f(\theta, \varphi)$ and differential cross section. ii) Derive simultaneous eigenvalue spectrum for J^2 and j_z.	07 07
		OR	
	a)	i) Why Born approximation is known as *high energy approximation*? Within the Born approximation derive an expression for scattering amplitude. ii) Obtain scattering cross section for (A) screened Coulomb potential and (B) hard sphere potential.	07 07
	b)	Attempt any three questions. i) Write criterion for validity of Born approximation. ii) Give difference between Born approximation and partial wave analysis. iii) Obtain the operation of $(s_- s_+)$ on *spin-up* wave function α. iv) For Pauli's matrices, obtain the value of $\sigma_x\sigma_x\sigma_x$.	03