M. Sc. Semester-III (MATHEMATICS)
Paper 12: Analytic Number Theory-I

MARCH- 2017
Code- 3473

Marks:70

Q-1. (a) Prove that $\sum_{d|n}\mu(d) = \left[\frac{1}{n}\right]$, for all positive integer n. [7]

(b) Define: Dirichlet product $*$. Show that it is commutative and associative. [7]

OR

Q-1. (a) Prove that $\sum_{d|n}\varphi(d) = n$, for all positive integer n. [7]

(b) If f is an arithmetical function (a.f.) with $f(1) \neq 0$, then show that there exists an a.f. g such that $f * g = I$. [7]

Q-2. (a) Prove that $\Lambda(n) + \sum_{d|n}\mu(d)\log d = 0$, for all positive integer n. [7]

(b) Find the Bell series of the Euler's function λ. [7]

OR

Q-2. (a) Find the Bell series of the Euler's function φ. [7]

(b) State and prove Selberg Identity. [7]

Q-3. (a) For all $x \geq 1$, show that $\sum_{n\leq x}\frac{1}{n} = \log x + C + O(\frac{1}{x})$, where C is Euler's constant. [7]

(b) For all $x \geq 1$, show that $\sum_{n\leq x} d(n) = x\log x + (2C-1)x + O(\sqrt{x})$. [7]

OR

Q-3. (a) For all $x \geq 1$, show that $\sum_{n\leq x}\frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \zeta(s) + O(x^{-s})$, where $s > 0$, $s \neq 1$. [7]

(b) For all $x \geq 2$, show that $\sum_{2\leq n\leq x}\frac{1}{n\log n} = \log(\log x) + K + O(\frac{1}{x\log x})$, where K is a constant. [7]

Q-4. (a) For all $x \geq 2$, show that $\sum_{\substack{p,\, prime \\ p \leq x}} \left[\frac{x}{p}\right] \log p = x \log x + O(x)$. [7]

(b) For all $x \geq 1$, show that $\left|\sum_{n \leq x} \frac{\mu(n)}{n}\right| \leq 1$. [7]

OR

Q-4. State and prove Abel's identity. [14]

Q-5. Prove: the relation $M(x) = o(x)$ as $x \to \infty$ implies $\psi(x) \sim x$ as $x \to \infty$. [14]

OR

Q-5. Let $A(x) = \sum_{n \leq x} \frac{\mu(n)}{n}$. Prove that the relation $A(x) = 0(x)$ as $x \to \infty$ implies the prime number theorem. [14]

—x—x—