November - 2015

M.Sc. (Mathematics) Semester – 3

Paper – 11: Combinatorial Analysis – I (Code: 3472)

Time: 2 ½ Hours **Total Marks: 70**

Q.1 (a) Derive the formula for the number of r – selection of an n – set. [7]

(b) Show that the number of possible subsets of an n – set is 2^n. [7]

OR

Q.1 (a) Prove: $\sum_{k=1}^{n} k^2 \binom{n}{k} = n(n+1)2^{n-2}$. [10]

(b) Give a combinatorial proof of $\binom{n}{k} = \binom{n}{n-k}$. [4]

Q.2 (a) Verify the following result using Inclusion – Exclusion principle: [7]

$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_1 \cap A_3| + |A_1 \cap A_2 \cap A_3|$

(b) Find the number of integers $a \in \{1, 2, 3, \ldots\ldots\ldots, 7000\}$ such that $\gcd\{a, 7000\} = 1$. [7]

OR

Q.2 (a) Prove in usual notations: $\varphi(n) = n\prod_{p/n}\left(1 - \frac{1}{p}\right)$ where $n \in N$ and p is prime. [10]

(b) Write down the formula for the number of derangements on n numbers $1, 2, 3, \ldots\ldots\ldots, n$ and verify it for $n = 1, 2, 3$ and 4. [4]

Q.3 (a) Derive the formula for Menage numbers. [10]

(b) Show that $n! = \sum_{k=0}^{n-1}(-1)^k \binom{n}{k} (n-k)^n$ [4]

OR

Q.3 (a) Let $f(n, k)$ denote the number of ways of selecting k objects, no two consecutive from n objects arranged in a row then prove that $f(n, k) = \binom{n-k+1}{k}$. [7]

(b) Find permanent of the following matrices using the method of permutations: [7]

$$(i)\begin{bmatrix} 2 & -1 & 1 & 3 \\ 4 & -2 & -1 & 1 \end{bmatrix} \qquad (ii)\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Q.4 (a) If a chess board C is partitioned into two independent parts D and E then prove that $R(x, C) = R(x, D) \cdot R(x, E)$. [7]

(b) Solve the recurrence relation: [7]

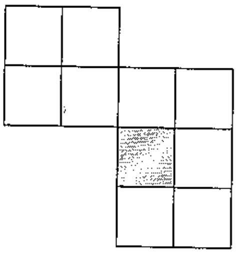
$a_n = 6a_{n-1} - 9a_{n-2}$ with $a_0 = 2, a_1 = 3$.

OR

Q.4 (a) Solve the recurrence relation: [7]

$-a_n = 3a_{n-1} - 2a_{n-2}$ with $a_0 = 2, a_1 = 1$.

(b) Find the rook polynomial for the following chess board: [7]



Q.5 (a) State and prove Ramsey's theorem for two components. [10]

(b) Show in usual notations $N(q, r; r) = q$ [4]

OR

Q.5 Prove in usual notations: [14]

(i) $N(q_1, q_2, \dots\dots\dots, q_t; 1) = q_1 + q_2 + \dots\dots\dots + q_t - t + 1$

(ii) $N(r, q; r) = q$.

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