

M. Sc. (Statistics) (Semester - III) Examination
Dec-2016 Code-3575
 Paper – X : Linear Models & Design of Experiments

Time : 2 hours

Marks: 70

- Q1 a) Let $Y_i, i = 1, 2, \dots, 6$ are independent variates with common variance σ^2 and $E(Y_1) = \theta_1 + \theta_4, E(Y_2) = \theta_2 + \theta_4, E(Y_3) = \theta_3 + \theta_5, E(Y_4) = \theta_4 + \theta_5, E(Y_5) = \theta_1 + \theta_6, E(Y_6) = \theta_2 + \theta_6.$ 8

i) Show that $\theta_1 - \theta_2$ is estimable. Find its BLUE and the variance of the BLUE.

Show that $\theta_1 + \theta_2$ is not estimable.

- b) Let Y_1, Y_2 and Y_3 are independent random variables with a common variance σ^2 and $E(Y_1) = \theta_1 + \theta_2, E(Y_2) = \theta_2 + \theta_3, E(Y_3) = \theta_1 + \theta_3.$ Prove that $b_1\theta_1 + b_2\theta_2 + b_3\theta_3$ is estimable if and only if $b_3 = b_1 + b_2$ and under this condition, obtain the Best Linear Unbiased Estimator of $b_1\theta_1 + b_2\theta_2 + b_3\theta_3.$ 6

OR

- Q1 a) State and prove Gauss – Morkoff's Theorem on linear estimation . 8
 b) Prove that necessary and sufficient condition for $b'\beta$ to be estimable is $\text{Rank}(A') = \text{Rank}(A', b).$ 6

- Q2 a) Define the following terms in Designs: 8
 i) Connectedness,
 ii) Balancing,
 iii) Orthogonality ,
 iv) Incidence Matrix and
 b) In usual notation prove that 6
 (i) $V(Q) = \sigma^2 C$
 (ii) $V(P) = \sigma^2 D$
 (iii) $\text{Cov}(P, Q) = -\sigma^2 D$

OR

- Q2 a) Prove that necessary and sufficient condition for a block design to be balanced is that all the non zero eigen roots OF C matrix are equal.. 8
 b) Check whether the block design with incidence matrix N 6

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$
 is connected, balanced and orthogonal..

- Q3 a) Define Balanced Incomplete Block Design. With usual parameters of B.I.B.D, prove that- 9
 i) $bk = vr$,
 ii) $r(k - 1) = \lambda (v - 1)$, and
 iii) $b \geq v$.
 b) State and prove necessary and sufficient condition for a block design to be orthogonal. 5

OR

- Q3 a) Derive the Intra-block analysis of a BIBD. 9
 b) Compare the efficiency of B.I.B.D. with R.B.D. 5
 Q4 a) For a Symmetric BIB Design (v, r, λ) . Prove that- 6
 If v is even, $(r - \lambda)$ must be a perfect square.
 b) Construct a BIBD with $v=9$, $b=12$, $r=4$, $k=3$, $\lambda = 1$ using mutually orthogonal latin square design.. 8

OR

- Q4 a) Give complementary design, derive design and residual design of following BIBD. 9
 $(1,3,4,5,9) (2,4,5,6,10) (3,5,6,7,11) (4,6,7,8,1) (5,7,8,9,2) (6,8,9,10,3)$
 $(7,9,10,11,4) (8,10,11,1,5) (9,11,1,2,6) (10,1,2,3,7) (11,2,3,4,8)$
 b) Construct a BIBD with parameters:- $V = b = 7$, $r = k = 3$, $\lambda = 1$ using elements of GF (7) 5
 Q5 a) Define the terms:- 6
 i) Symmetric Factorial Design,
 ii) Partial and Total confounding,
 iii) Generalized Interactions.
 b) Explain main effect and interaction for 3^2 design. 8

OR

- Q5 a) Define the device of 'confounding' and discuss the Yates procedure to analyse the 2^n factorial design.. 9
 b) Explain the method of confounding 2 independent interaction in 2^m design. 5