13 APR 2019

Code-3580

M. Sc. Statistics (Sem.-IV) Examination

Time: 2.30 Min] Paper 15: Decision Theory Reliability & Industrial Statistics [Marks: 70

1(a) Define: Complete class of decision rule, Minimal complete class of decision rule. Non randomized decision rule and risk function. 4

(b) Let X be a Binomial variable with p.m.f. $f(x \mid \theta) = 2C_x\ \theta^x\ (1-\theta)^{2-x}$, $x = 0,1$. Parametric space $\cong$ $\{ \theta_1 = 1/3, \theta_2=2/3 \}$ and action space = $\{ a_1 , a_2 \}$ and loss function is 10

	a_1	a_2
θ_1	4	1
θ_2	-2	0

1) Obtain the set of non randomized rules.
2) Risk function for all non randomized decision rule
3) Obtain Minimax decision rule
4) Set of admissible decision rule

OR

1(a) 1. Show that the problem of estimation is statistical decision problems. 4

(b)) Consider a decision problem(Ω, A, D) with $\Omega=\{ \theta_1, \theta_2\}$, $A = \{a_1, a_2\}$ and Sample space $X = \{x_1, x_2, x_3\}$ and the loss function and probability mass function of X are given in the following tables. 10

	$P(X=x_1)$	$P(X=x_2)$	$P(X=x_3)$
$\theta = \theta_1$	0.3	0.6	0.1
$\theta = \theta_2$	0.2	0.3	0.5

	a_1	a_2
θ_1	-7	14
θ_2	20	-5

Obtain class of all possible non-randomized decision rules. Find the risk of the rule.

2.(a) A random sample of size n is taken from $N(\mu, 4)$. Verify whether sample mean is a Bayes rule? Is it true for any type of prior of μ over $(-\infty, \infty)$. Justify your answer. 7

(b) Define a Bayes decision rule. 7

Let Θ = Action space = $(0,\infty)$ and $L(\theta,a)=(\theta-a)^2$. Let the distribution of X be given by $f(x|\theta) = 1/\theta$ if $0 < x < \theta$ and $= 0$ otherwise. Find Bayes rule and Bayes Risk with respect to prior distribution $g(\theta) = \theta\exp(-\theta)$ if $\theta > 0$ and $= 0$ otherwise.

OR

2.(a) Define:(i) Prior distribution (ii) Bayes rule 4

(iii) Posterior distribution (iv) Extended Bayes rule.

(b) Let parametric space be R^+, $L(\theta,a) = (\theta-a)^2$ and distribution of X be poisson with parameter $\theta>0$, 10

find Bayes rule with respect to prior distribution gamma
(α, β) Also determine Bayer risk and Limit Bayes Rule.

3. (a) Define Series and parallel systems. Obtain reliability of both the systems. Give your conclusion. 7

(b) What is bridge structure? Discuss the method to obtain reliability of a bridge structure. 7

OR

3. (a) Define reliability, cumulative hazard function. Obtain an expression for pdf of a life time model using the given hazard rate. 7

(b) Find the reliability function, hazard function and mean time to failure (MTTF) of the life time model $f(x) = (\alpha/\beta)\,(x/\beta)^{\alpha-1} \exp\left(-(x/\beta)^{\alpha}\right)$, $x,\alpha,\beta>0$. 7

4 (a) Discuss the maximum likelihood method of estimation for Weibull distribution based on a failured censored sample.

(b) Discuss type-II censoring. Under this scheme obtain MLE of reliability of a unit having exponential life time model with mean θ.

OR

4 (a) Failure rate function h(t) of Life time distribution is given as following. Hence obtain reliability functions, probability density functions and MTTF $h(t) = 1/\theta$, $t>0$. $\theta>0$.

(b) Explain k out n system derive the expression for reliability of this system. Hence obtain the reliability of TMR system for R=1/2.

5(a) Describe double sampling plan procedures for attributes and find out the expression for OC function.

(b) Describe sequential sampling plan by attributes. How do you construct O.C. function for this plan?

OR

5(a) Describe single sampling plan for variables when the quality characteristic is assumed to follow normal distribution. Obtain its O.C function when upper specification limit is known and process standard deviation is also known.

(b) Discuss the differences between sampling plans for attributes and variables.